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An economic order quantity model with partial backordering and incremental discount

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ABSTRACT

Determining an order quantity when quantity discounts are available is a major interest of material managers. A supplier offering quantity discounts is a common strategy to entice the buyers to purchase more. In this paper, EOQ models with incremental discounts and either full or partial backordering are developed for the first time. Numerical examples illustrate the proposed models and solution methods.

1. Introduction and literature review

Since Harris (1913) first published the basic EOQ model, many variations and extensions have been developed. In this paper we combine two of those extensions: partial backordering and incremental quantity discounts.

Montgomery, Bazaraa, and Keswani (1973) were the first to develop a model and solution procedure for the basic EOQ with partial backordering (EOQ–PBO) at a constant rate. Others taking somewhat different approaches have appeared since then, including Pentico and Drake (2009), which will be one of the two bases for our work here. In addition, many authors have developed models for the basis EOQ-PBO combined with other situational characteristics, such as Wee (1993) and Abad (2000), both of which included a finite production rate and product deterioration, Sharma and Sadiwala (1997), which included a finite production rate with yield losses and transportation and inspection costs, San José, Sicilia, and García-Laguna (2005), which included models with a non-constant backordering rate, and Taleizadeh, Wee, and Sadjadi (2010), which included production and repair of a number of items on a single machine. Descriptions of all of these models and others may be found in Pentico and Drake (2011).

Enticing buyers to purchase more by offering either all-units or incremental quantity discounts is a common strategy. With the all-units discount, purchasing a larger quantity results in a lower unit purchasing price for the entire lot, while incremental discounts only apply the lower unit price to units purchased above a specific quantity. So the all-units discount results in the same unit price for every item in the given lot, while the incremental discount can result in multiple unit prices for an item within the same lot (Tersine, 1994). In the following we focus on the research using only an incremental discount or both incremental and all-units discounts together. Since Benton and Park (1996) prepared an extensive survey of the quantity discount literature until 1993, we will describe newer research, along with a short history of incremental discounts and older research which is more related to this paper.

The EOQ model with incremental discounts was first discussed by Hadley and Whitin (1963). Tersine and Toelle (1985) presented an algorithm and a numerical example for the incremental discount and examined the methods for determining an optimal order quantity under several types of discount schedules. Güder, Zydiak, and Chaudhry (1994) proposed a heuristic algorithm to determine the order quantities for a multi-product problem with resource limitations, given incremental discounts. Weng (1995) developed different models to determine both all-units and incremental discount policies and investigated the effects of those policies with increasing demand. Chung, Hum, and Kirca (1996) proposed two coordinated replenishment dynamic lot-sizing problems with both incremental and all-units discounts strategies. Lin and Kroll (1997) extended a newsboy problem with both all-units and incremental discounts to maximize the expected profit subject to a constraint that the probability of achieving a target profit level is no less than

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a predefined risk level. Hu and Munson (2002) investigated a dynamic demand lot-sizing problem when product price schedules offer incremental discounts. Hu, Munson, and Silver (2004) continued their previous work and modified the Silver-Meal heuristic algorithm for dynamic lot sizing under incremental discounts. Rubin and Benton (2003) considered the purchasing decisions facing a buying firm which receives incrementally discounted price schedules for a group of items in the presence of budgets and space limitations. Rieksts, Ventura, Herer, and Sun (2007) proposed a serial inventory system with a constant demand rate and incremental quantity discounts. They showed that an optimal solution is nested and follows a zero-inventory ordering policy. Haksever and Moussourakis (2008) proposed a model and solution method to determine the ordering quantities for multi-product multi-constraint inventory systems from suppliers who offer incremental quantity discounts. Mendoza and Ventura (2008) incorporated quantity discounts, both incremental and all-units, on the purchased units into an EOQ model with transportation costs. Taleizadeh, Niaki, and Hosseini (2009) developed a constrained multi-product bi-objective single-period problem with incremental discounts and fully lost-sale shortages. Ebrahim, Razm, and Haleh (2009) proposed a mathematical model for supplier selection and order lot sizing under a multiple-price discount environment in which different types of discounts including all-unit, incremental, and total business volume are considered. Taleizadeh, Niaki, Aryanezhad, and Fallah-Tafti (2010) developed a multi-products multi-constraints inventory control problem with stochastic period length in which incremental discounts and partial backordering situations are assumed. Munson and Hu (2010) proposed procedures to determine the optimal order quantities and total purchasing and inventory costs when products have either all-units or incremental quantity discount price schedules. Bai and Xu (2011) considered a multi-supplier economic lot-sizing problem in which the retailer replenishes his inventory from several suppliers who may offer either incremental or all-units quantity discounts. Chen and Ho (2011) developed an analysis method for the single-period (newsboy) inventory problem with fuzzy demands and incremental discount. Taleizadeh, Barzinpour, and Wee (2011) discussed a constrained newsboy problem with fuzzy demand, incremental discounts, and lost-sale shortages. Taleizadeh, Niaki, and Nikousokhan (2011) developed a multi-constraint joint-replenishment EOQ model with uncertain unit cost and incremental discounts when shortages are not permitted. Bera, Bhunia, and Maiti (2013) developed a two-storage inventory model for deteriorating items with variable demand and partial backordering. Lee, Kang, Lai, and Hong (2013) developed an integrated model for lot sizing and supplier selection and quantity discounts including both all units and incremental discounts. Archetti, Bertazzi, and Speranza (2014) studied the economic lot-sizing problem with a modified all-unit discount transportation cost function and with incremental discount costs.

According to the above mentioned research, it is clear that no researchers have developed an EOQ model with partial backordering and incremental discounts. Taleizadeh and Pentico (2014) developed an EOQ model with partial backordering and all-units discounts. In this paper we develop EOQ models with fully and partially backordered shortages when the supplier offers incremental discounts to the buyer.

2. Model development

In this section we model the defined problem under two different conditions: full backordering and partial backordering. But first we briefly discuss the EOQ model with full or partial backordering when discounts are not assumed. We use the following notation.

Parameters

A	Fixed cost to place and receive an order
β	The fraction of shortages that will be backordered
C_j	The purchasing unit cost at the jth break point
D	Demand quantity of product per period
g	The goodwill loss for a unit of lost sales
i	Holding cost rate per unit time
n	Number of price breaks
q_j	Lower bound for the order quantity for price j
P	Selling price of an item
π	Backorder cost per unit per period
π'_j	The lost sale cost per unit at the jth break point of unit purchasing cost, $\pi'_j = P - C_j + g > 0$

Decision variables

B	The back ordered quantity
F	The fraction of demand that will be filled from stock
Q	The order quantity
T	The length of an inventory cycle

Dependent variables

ATC	Annual total cost
ATP	Annual total profit
CTC	Cyclic total cost
CTP	Cyclic total profit

2.1. EOQ models with no discount

In this section we briefly discus EOQ models with fully or partially backordered shortages when discounts are not available. For the first case, the EOQ models with fully backordered shortages (see Fig. 1), Pentico and Drake (2009) derived the optimal values of F and T as:

$$F^* = \frac{\pi}{\pi + iC} \tag{1}$$

$$T^* = \sqrt{\frac{2A}{iCD}}\sqrt{\frac{\pi + iC}{\pi}} \tag{2}$$

For the second case, the EOQ model with partial backordering, Pentico and Drake (2009) showed that the values of F and T that minimize annual total cost are

$$F^* = \frac{(1-\beta)\pi' + \beta\pi T^*}{(iC + \beta\pi)T^*} \tag{3}$$

$$T^* = \sqrt{\frac{2A}{iCD}\left[\frac{iC + \beta\pi}{\beta\pi}\right] - \frac{[(1-\beta)\pi']^2}{\beta iC\pi}} \tag{4}$$

only if β is at least as large as a critical value β' given by Eq. (5)

$$\beta' = 1 - \frac{\sqrt{2AiCD}}{D\pi'} \tag{5}$$

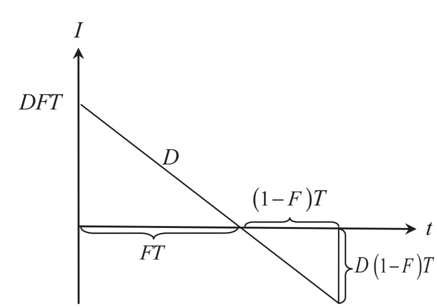

Fig. 1. EOQ model with fully backordered shortages.