Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 129 (2016) 10–30
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original articles

# A differential evolution algorithm for yield curve estimation

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Received 19 February 2015; received in revised form 25 February 2016; accepted 11 April 2016
Available online 25 May 2016

**Abstract**

Modeling the term structure of government bond yields is of great interest to macroeconomists and financial market practitioners. It is crucial for bonds and derivatives pricing, risk management, and reveals market expectations, which is essential for monetary policy decisions. This paper suggests the use of a differential evolutionary algorithm to estimate yield curves for US Treasury bonds. It considers parsimonious modeling to avoid non-convergence and high instability of traditional optimization algorithms when estimating model parameters caused by the choice of their initial values during curve fitting. In this approach, the whole yield curve for different maturities is obtained by models parameters estimates. Computational experiments show that the differential evolutionary algorithm provides more accurate yield curves than the ones derived by nonlinear least squares and genetic algorithm approaches.


*Keywords:* Yield curve; Differential evolution; Optimization; Interest rate; Finance

## 1. Introduction

The yield curve displays the connection between the nominal interest rates of default zero-coupon securities and their term to maturity. It is an essential tool in finance and macroeconomics. In finance, the term structure of interest rates is employed in risk management, portfolio selection, financial corporative decisions and, mainly, in financial bond pricing [35]. In macroeconomics, the yield curve is a key element in monetary decision making, as well as a measure of market expectations of future interest rates, inflation, and economic activity, given the current market conditions [59].

Yield curve estimation basically is a nonlinear parameter optimization problem within the framework of parsimonious modeling. The information behind the interest rate term structure reflects the true market status and expectations. Thus its estimation should be as accurate as possible. The function developed by [44] and its augmented version, proposed by [57], are the most important parsimonious yield curve models used by major central banks that report their estimations to the Bank for International Settlements [3].

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http://dx.doi.org/10.1016/j.matcom.2016.04.004

Proposals of simple functional forms for representing a yield curve have received considerable attention in the literature [30,4,23]. However, the issue of the algorithms to estimate the parameters of the models is often neglected. The use of mathematical programming algorithms to solve highly nonlinear optimization problems have been limited by non-convergence (i.e., the results may not correspond to a global optimum), and high sensitive results to the initialization of the algorithms [27]. For example, [14,4,30] analyzed the algorithms currently adopted in yield curve parameter estimation. More recently, [42] addressed the use of a wide range of nonlinear optimization algorithms to estimate parameters of yield curves from actual bond market data. The authors conclude that because of the nature of the problem, certain classes of algorithms are more effective than others. They also highlight the fact that slight differences in the estimated curves can result in significant differences when pricing bonds or portfolios [42].

The use of genetic algorithms (GA) to estimate the term structure of interest rates in the Spanish bond market was suggested in [27]. The authors show that the GA approach reduces the risk of false convergence and provides stable parameters without imposing arbitrary restrictions. [22] uses GA and splines to estimate the yield curve and found similar results. These studies suggest that methods based on a computational intelligence and evolutionary computation techniques may also be effective as a tool for yield curve modeling.

Evolutionary computation methods are one of the important components of computational intelligence. They are particularly valuable to handle complex search spaces often found in highly nonlinear optimization problems. Based on the underlying relation between optimization and biological evolution, evolutionary computation uses iterative progress of a population of candidate solutions selected in a guided random search with implicit parallelism [16]. The field of evolutionary computation and algorithms (EA) includes genetic algorithms (GA), evolutionary programming (EP), evolution strategies (ES), genetic programming (GP), and differential evolution (DE).[1]

Applications of evolutionary computation in economics and finance are generally associated with GP and GA [53]. For example, a dynamic proportion portfolio insurance strategy using GP and principal component analysis to build an equation tree for the risk multiplier was proposed by [8]. They show that their approach gives more profitable solutions than the proportion portfolio insurance strategy. As an extension of the GP, [12] developed a genetic network programming technique with control nodes aiming at portfolio optimization of the Japanese stock market. They conclude that the extended GP model outperforms traditional models in terms of accuracy and efficiency. [46,21] also exploits GP for the stock trading and bankruptcy prediction, respectively.[2]

Because the less restrictive assumptions about the parameter space required, GA are a widely used tool in financial optimization, especially for portfolio optimization [32,38,45,37]. [6] uses GA in portfolio optimization considering different risk measures. Portfolio selection considering minimum transaction lots, cardinality constraints, and sector capitalization was also tackled by [54] using GA. [28] developed a model that includes cost dependency for portfolio selection and their results reveal that GA perform better than an electromagnetism-inspired algorithm. A genetic relation algorithm with guided mutation was derived by [11] to solve large-scale portfolio optimization. The effectiveness of five state-of-the-art multi-objective EA and a steady-state EA for the mean–variance cardinality-constrained portfolio optimization problem has been compared by [1].[3] In addition, GA have been successfully used in portfolio value-at-risk forecasting [36], stock market trading [20], in examining the relationship between wealth dynamics and risk preferences in multi-asset artificial stock market evaluations [9], and in financial portfolio management using technical analysis indicators [29].

The DE algorithm was introduced by [55]. Unlike standard EA, DE uses the difference of candidate solutions to search for the extrema of real valued functions. Compared against alternative evolutionary algorithms, the features that turn the DE algorithm attractive to solve optimization problems are its simplicity of implementation[4]; its high accuracy and robustness to deal with distinct classes problems, such as separable, non-separable, modal, and multimodal functions [64,60,10,31,13]; very few control parameters (three in classical DE)[5]; and, finally, for expensive and large-scale optimization problems, its space complexity is lower than of most EA.

[1] In all evolutionary computation approaches there are different algorithms available, which mainly differ in terms of its operators. Recently, [34] provides a comparison of different evolutionary algorithms in several case studies.

[2] Bankruptcy prediction modeling was also evaluated using GA by [52].

[3] For a survey of metaheuristics in portfolio optimization, see [58].

[4] Methods based on particle swarm optimization are also straightforward to implement, but [15,50] note the largely superior performance of DE in certain varieties of problems.

[5] Studies such as that of [49] show that adaptation rules for some control parameters improve the performance of the DE algorithm without affecting its computational complexity.