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Some single-machine scheduling problems with elapsed-time-based and position-based learning and forgetting effects

Chia-Huang Wu[a], Wen-Chiung Lee[a], Peng-Jen Lai[b], Jen-Ya Wang[c,*]

[a] *Department of Statistics, Feng Chia University, Taichung 40724, Taiwan, ROC*

[b] *Department of Mathematics, National Kaohsiung Normal University, Kaohsiung 82446, Taiwan, ROC*

[c] *Department of Computer Science and Information Management, Hungkuang University, Taichung 43302, Taiwan, ROC*

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ABSTRACT

In this paper, a generalized model with past-sequence-dependent learning and forgetting effects is proposed. Both effects are assumed to be dependent on the sum of processing time as well as the scheduling position. Based on this model, we investigate and prove that some single-machine problems remain polynomially solvable with certain agreeable conditions. We further show that many models known in the literature are special cases of our proposed model. Several helpful lemmas are presented to analyze single-machine scheduling problems with various objective functions: makespan, total completion time, weighted completion time, and maximum lateness.

1. Introduction

In production and manufacturing, when a common facility is used to produce different items, learning and forgetting effects are always presented. These effects are particularly prominent if the facility is labor-oriented and not completely automated. When goods are produced in a batch environment, worker's productivity commonly increases because of the repeated processing and experience accumulation. Based on the learning effect, the workers become skilled in handling raw materials and components, operating software, and controlling machines. On the other hand, for products with short cycle times, workers must learn various skills in a short period of time without repeated practice. In this situation, a loss of learning, referred to as forgetting, may take place then cause decay in productivity. Biskup [1] first introduced single-machine scheduling problems with learning effects. In addition, Biskup [2], Janiak and Rudek [3] presented a comprehensive survey of scheduling problems with learning effects.

* Correspondence to: No. 1018, Sec. 6, Taiwan Boulevard, Taichung, 43302, Taiwan, ROC. Tel.: +886 4 26348652x5417.
E-mail address: jywang@sunrise.hk.edu.tw (J.-Y. Wang).

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Recently, various learning effect models have been investigated by many researchers. Lee et al. [4] studied the makespan minimization problem based on a single-machine scheduling problem with learning effects and release times. Lee [5] offered a simple and realistic learning effect model which has the flexibility to describe different learning curves easily. Bai et al. [6] considered the minimization problems of various objective functions based on a general exponential learning effect model. The actual processing time of a job is defined by its position in a sequence and an exponential function of weighted cumulative processing time of past jobs where the weights are position-dependent. Based on the model of Biskup [1], Behdin et al. [7] presented a new binary integer programming model to obtain near-optimal schedules for a single machine problem with multiple availability constraints. Jiang et al. [8] released the assumption of the learning coefficient in the model of Kuo and Yang [9], and showed that the makespan and sum of weighted completion time minimization problems are both NP-hard. In [10], the influence of the sum of processing times on the total weighted completion time was emphasized. This model can model both learning and aging (deteriorating) effects. Moreover, such learning effects can also be found in other environments. For example, Janiak and Rudek [11] introduced an interesting makespan minimization problem on a single machine which takes place in a computer network. Similar single-machine problems with learning effects can be found in other literature [12–16].

Beside the learning effect models mentioned earlier, the learning effect models with deterioration effect also have captured considerable attention in recent years. Cheng et al. [17] gave a detailed review of the scheduling problems with deteriorating jobs. Lately, Ji and Cheng [18] considered a single-machine scheduling problem with an availability constraint where the processing time is a simple linear deteriorating function of its waiting time. Yang [19] investigated group scheduling problems with learning and deterioration effects on a single-machine setting. The makespan minimization problem based on this model has been shown to be polynomially solvable. Lee and Lai [20] proposed a generalized model with learning and deterioration effects and provided the optimal solutions for some single-machine problems. Wu et al. [21] presented a single-machine model with both learning and deterioration effects and provided some optimal rules for various objective functions. Yin et al. [22] proposed a revision of the model of Lee and Lai [20]. Bai et al. [23] considered a single-machine group scheduling problem with general learning and deterioration effects. Based on the model of Lee and Lai [20], Wang and Wang [24] showed that some single-machine problems are polynomially solvable under certain conditions. Wang et al. [25] investigated the effects of deterioration and learning on single machine problem with various objective functions.

Similar to the deterioration effect, in practice, the forgetting effect may occur along with the learning effect. Wixted and Ebbesen [26], and Jaber et al. [27] pointed out that learning and forgetting are mirror images of each other. As suggested in [28–30] as opposed to learning, the forgetting effects reduce the production rate, influence the product quality and increase the setup cost or the penalty of production breaks. Yang and Chand [31] constructed three basic models to investigate the impacts of learning and forgetting effects on single-machine problems to minimize the total completion time with family setup time. Kim and Nembhard [32] explored the effects of several exogenous and controllable factors on learning and forgetting behaviors of workers. Nembhard and Bentefouet [33] developed a mixed integer linear program to solve the parallel system scheduling problem with learning and forgetting effects. Lai and Lee [34] proposed a single-machine model with both learning and forgetting effects. They also presented an example to illustrate that the forgetting effect might exist in some situations.

As mentioned earlier, the research of the coexistence of the effects of learning and forgetting is limited. Moreover, learning and forgetting effects should be not only time-dependent but also position-dependent. That is, learning and forgetting effects on a job should be a function of the cumulative effects of elapsed time for various positions. This motivates us to propose a model with both learning and forgetting effects where both effects are elapsed-time-based and position-based.

The rest of this paper is organized as follows. The notations and problem formulation are given in the next section. In Section 3, several properties are shown to deal with the single machine problems. We develop