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On the product portfolio planning problem with customer–engineering interaction

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ABSTRACT

Jiao and Zhang (2005) propose a shared-surplus maximization model considering customer preferences and choice behavior as well as platform-based product costing to tackle the so-called product portfolio planning problem (optimal mix of products and attributes). They emphasize the joined consideration of customers concerns and operational implications and propose a stochastic, mixed-integer, non-linear program. We discuss several issues and ambiguities of the original approach and propose some improvements such as demand model calibration, deterministic customer surplus, and an effective objective function.

1. Introduction

[7] propose a shared-surplus maximization model considering customer preferences and choice behavior as well as platform-based product costing to tackle the so-called product portfolio planning problem. For the solution of the problem a heuristic genetic algorithm procedure is applied. The product portfolio planning problem is the selection of an optimal mix of products and product attributes offered to customers. Therefore, the authors emphasize the joined consideration of customer concerns and operational implications, i.e., the objective is a so-called shared surplus to leverage both the customer and engineering concerns. The shared surplus intends to account for the consumer surplus and the producer surplus at the same time. The consumer surplus, i.e., the amount that customers benefit by being able to purchase a product for a price that is less than that they would be willing to pay, is measured using consumers utilities and choice probabilities given by the multinomial logit model (MNL). The MNL is the workhorse in discrete choice analysis for decades [8,2,9,1]. The producer surplus, i.e., the amount that producers benefit by selling at a price that is higher than that they would be willing to sell for, is measured by consumers choice probabilities and by the so-called process capability index (PCI) which is based on the expected cycle time [6]. For a more detailed discussion of the fundamental issues of the product portfolio planning problem we refer to [7].

In this contribution we discuss several issues and ambiguities found in the approach by [7]. We propose some selected improvements to their approach. We focus on customer preferences and choice probabilities (Section 2) and the resulting mathematical programming formulation (Section 3). In particular, in contrast to the shared surplus concept by [7], we propose to choose a producer surplus maximization principle to come up with useful product portfolio planning decisions. Further, we propose improvements to demand model calibration and the formulation of deterministic customer surplus.

2. Customer preferences and choice probabilities

[7] denote

- I segments of customers,
- J products of the company,
- N all products available (i.e., $J \subseteq N$) with $N \setminus J$ denoting products of competitors (this might include the choice option of choosing not to purchase any product),
- K attributes (with $K + 1$ as the price attribute),
- L_k discrete levels of attribute k,

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and define the utility of segment i for product j as

$$U_{ij} = \sum_{k=1}^{K} \sum_{l=1}^{L_k} \left(w_{jk} u_{ikl} x_{jkl} + \pi_j \right) + \epsilon_{ij}, \tag{1}$$

with

- u_{ikl} part-worth utility of customer segment i and the lth level of attribute k,
- w_{jk} utility weight of attribute k for product j (coefficient of the utility function),
- π_j constant utility contribution for product j (coefficient of the utility function), and
- ϵ_{ij} error term for customer segment i and product j.

The binary decision variable

$$x_{jkl} = \begin{cases} 1 & \text{if the } l\text{th level of attribute } k \text{ is contained in product } j, \\ 0 & \text{otherwise.} \end{cases}$$

Of course, we might consider set K_j (and set L_{kj}) indicating that some attributes are only available for certain products. However, we keep our nomenclature as close as possible to [7] for the sake of comparability.

In discrete choice analysis, π_j is referred to as the alternative specific constant [15, p. 20]. The alternative-specific constant for product j captures the average effect on utility of missing customer characteristics and of all product attributes that are not included in $\{1, \ldots, K+1\}$. Thus, it serves a similar function to the constant (intercept) in a regression model, which also captures the average effect of all non-included factors. Therefore, (1) is written as

$$U_{ij} = \pi_j + \sum_{k} \sum_{l=1}^{L_k} \left(w_{jk} u_{ikl} x_{jkl} \right) + \epsilon_{ij}. \tag{2}$$

For simplicity reasons we write the utility function as

$$U_{ij} = \Psi_{ij} + \epsilon_{ij} \tag{3}$$

with

$$\Psi_{ij} = \pi_j + \sum_{k} \sum_{l=1}^{L_k} w_{jk} u_{ikl} x_{jkl} \tag{4}$$

denoted as the deterministic part of utility U_{ij}. Hence, ϵ_{ij} denotes the stochastic part. Concerning the choice model and the product demand, [7] apply the utility maximization choice (decision) rule as mentioned in [2, pp. 37–38]. That is, consumers of segment i choose the product j that maximizes their utility:

$$U_{ij} > U_{in} \quad \forall\, n \in N,\ j \neq n. \tag{5}$$

Obviously, utility U_{ij} of (3) is a stochastic quantity. Thus, we can only make probability statements about the choice problem of (5):

$$P_{ij} = \text{Prob} \left(U_{ij} > U_{in}\, \forall\, n \in N,\ j \neq n \right) \tag{6}$$

P_{ij} denotes the choice probability for customers of segment i choosing product j. Concerning the choice probabilities, [7] assume ϵ_{ij} to be independent, identically extreme value distributed. Therefore, the choice problem of (6) can be written as

$$P_{ij} = \frac{\exp\left(\mu \Psi_{ij}\right)}{\sum_{n \in N} \exp\left(\mu \Psi_{in}\right)} \tag{7}$$

what is the well-known MNL. The algebraic transformations that lead from (6) to (7) can be found in [15, pp. 74–76]. $\mu > 0$ is a scale parameter, that is not identified in model estimation, i.e., the procedure to determine w_{jk} and π_j from empirical choice data [14,11,13]. Therefore, it is normalized to 1, in general. [7] denote the MNL as

$$P_{ij} = \frac{\exp\left(\mu U_{ij}\right)}{\sum_{n \in N} \exp\left(\mu U_{in}\right)}. \tag{8}$$

Yet, U_{ij} is stochastic, and hence (8) would be written as

$$P_{ij} = \int_{\epsilon_{ij}} \frac{\exp\left(\mu U_{ij}\right)}{\sum_{n \in N} \exp\left(\mu U_{in}\right)} f\left(\epsilon_{ij}\right) d\epsilon_{ij} \tag{9}$$

what is not the MNL, but a non closed form choice probability with a multi-dimensional integral to be simulated. Though, [7] do not give any justification for choice probabilities of the form of (8) (or rather Eq. (9)).

[7] propose to calibrate the MNL on actual market shares using post hoc optimization with respect to μ. But the reader should notice that a modification of μ yields a modification of the assumed variance of the stochastic part of utility. In fact, the original values of w_{jk} and π_j are weighted by the standard deviation of the stochastic part of utility. If μ is modified, w_{jk} and π_j must be modified accordingly [15, p. 24]. Further, to adjust market shares by a modification of μ is not effective, because multiplying each product's utility by a positive constant, i.e., μ, does not change the choices of the customers in a segment i: The product with the highest utility is the same no matter by which positive value utility is scaled.

Rather, a calibration towards actual market shares is done by adjusting the alternative-specific constant π_j properly [15, p. 33]. Let be

- Q_i size (i.e., number of consumers) of market/customer segment i,
- S_j the actual market share, i.e., the market share we want to adjust π_j to,
- δ an iteration counter,
- π_j^{δ} the alternative specific constant for iteration δ, with $\pi_j^0 = \pi_j$, and
- $\hat{S}_j^{\delta}$ the predicted market shares of the MNL of (7) given as $\hat{S}_j^{\delta} = \sum_i Q_i P_{ij}$ for iteration δ using π_j^{δ} in (4).

Then

$$\pi_j^{\delta} = \pi_j^{\delta-1} + \ln\left(S_j / \hat{S}_j^{\delta-1} \right) \tag{10}$$

is an adequate adjustment. π_j^{δ} is repeatedly adjusted until $\hat{S}_j^{\delta} \approx S_j$.

3. Mathematical programming formulation

[7] suggest to maximize the consumer surplus by the objective

$$\text{maximize} \quad \sum_{i=1}^{I} \sum_{i=j}^{J} \left(U_{ij} - p_j \right) P_{ij} Q_i \tag{11}$$

with the decision variable p_j as the price of product j. Note, $p_j = \sum_{l=1}^{L_{K+1}} u_{i,K+1,l}$ and $\sum_{l=1}^{L_{K+1}} x_{j,K+1,l} = 1$. Further, $u_{1,K+1,l} = u_{2,K+1,l} = \cdots = u_{I,K+1,l}$, because p_j implies that the price for product j is the same for all customer segments $i \in I$. The consumer surplus of (11) is questionable for several reasons. First, this approach demands to define utility (2) in monetary units. While this is feasible in formal terms it is in contrast to the basic idea of utility which says that utility has no specific unit [15, pp. 14–29]. Second, the optimal solution to (11) is in any case $p_j = 0$ (or, $p_j = -\infty$ in a theoretical perspective) for all $j \in J$. Even if U_{ij} is a function of p_j the corresponding price coefficient (i.e., $w_{j,K+1}$) is strictly smaller than zero, yielding $p_j = 0$. Third, U_{ij} is a random quantity (2), therefore (11) is a stochastic problem. If one is anxious to use the consumer