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On the non-emptiness of the one-core and the bargaining set of committee games

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ABSTRACT

We study the committee decision making process using game theory. Shenoy [15] introduced two solution concepts: the one-core and the bargaining set, and showed that the one-core of a simple committee game is nonempty if there are at most four players. We extend this result by proving that whether the committee is simple or not, as far as there are less than five players, the one-core is nonempty. This result also holds for the bargaining set.

1. Introduction

Our game model is the one considered by Shenoy [15] committee game that generalizes the voting model introduced by von Neumann and Morgenstern [17] under the name of simple game. A committee game is a tuple (N, A, v, u): N is the committee which is any finite group of persons who have to pick one option from the finite given set of outcomes A. It can be assumed that they are situated in one room. They might arrive at a collective decision after lengthy deliberations. v is the rule which specifies by which the committee arrives at a decision. The rule is designed such that the decision of the committee will consist of a unique outcome. For any coalition (nonempty group of players) S, $v(S)$ which is a (possibly empty) subset of A represents the set of outcomes on which S is powerful. More precisely, if an outcome x belongs to $v(S)$, then members of S altogether can enforce the election of x whatever the members of $N \setminus S$ do. The final outcome is either the initial status quo or an outcome enforced by a powerful coalition on that outcome. u is the payoffs vector or utilities vector, that indicates, for each committee member i his utility $u_i(x)$ at any outcome x. Instead of considering utilities vectors one could consider that each member of the committee has a preference relation which is a weak order on the set of all outcomes, thus yielding a preference profile. Utility is assumed to be nontransferable and interpersonal comparison of utilities is assumed to be meaningless. The vote takes place within a sequential framework: departing from an initial status quo a_0, either it is adopted as the final outcome or a player i introduces a motion say a_1 in the form of a proposal (i, a_1). The proposal is debated by the members. At the end of the debate there are four possible courses of action:

1. there exists a coalition S powerful on a_1 whose members ask for the adoption of a_1. In this case the game is over and the final decision is a_1;
2. player i withdraws his motion and no other motion is introduced. Then the game is over and the final decision is a_0;
3. player i withdraws his motion and another proposal is made and the process continues (as indicated in Fig. 1 below);
4. another member $j \in N$ introduces another motion $y \in A$ and the two proposals are put to vote with the members voting for one of the two motions. The motion that wins becomes the new proposal and the process continues. In the case neither motion gets a decisive vote, the motion introduced first is considered undefeated and remains as the current proposal, the current status quo.

The process above continues and the game ends once they arrive at an ultimate option a^* for which there exists a coalition S such that $a^* \in v(S)$ and members of which ask for the adoption of a^*. Such an option is said to be stable. Voters get their payoffs only when the final outcome is reached. Any member is allowed to

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suggest any alternative for consideration by the committee. In such a social choice context, the question generally asked is how a voter should behave or should vote when solicited to join a coalition in order to decide over a status quo. Another relevant issue is to determine what could be a suitable choice of a given member i if he is given the opportunity to introduce a motion.

The core is a solution concept in which any member is recommended to vote for x against y whenever he strictly prefers x to y (that is, $u_i(x)$ is greater than $u_i(y)$) if x is opposed to y. Furthermore, a committee member should propose an outcome x if it is his (or one of his) best element in the core. An outcome x belongs to the core if it is undominated, that is, there does not exist another outcome y, a coalition S powerful on y and all members of which are strictly better off at y than at x. This behavioral pattern of the core has been criticized by Shenoy [15] who argued that a player who is making a proposal does not cooperate in any effort to dominate the proposal. In other words, such a player cares about undominated outcomes via coalitions not containing him, and pick only maximal ones. This yields to the definition of the one-core. Unfortunately, the one-core might be empty even if players' preferences are strict. Indeed Shenoy [15] provided a 5-player game with an empty one-core. If the number of players is less than five, the one-core is nonempty, provided that the committee game is simple. However, a number of interesting real life instances of vote can be associated with committee games that are not simple. In the rule of k-names introduced by Barberà and Coelho [3], the set of deciders (voters) is divided into two groups: the proposers and the chooser. Proposers consider the set of all alternatives (candidates) to a position and screen k of them. Then, the chooser picks the appointee out of these k names. Thus, a coalition S may be powerful on an alternative x (in the case where the chooser is in S and x belongs to the k alternatives collectively selected by proposers) meanwhile S is not powerful on another alternative y.

Another example is the promotion of a faculty member in a given department. Consider a council in which the promotion of a faculty member is enforced if it is supported by a strict majority of committee members. A majority of members (made of assistant professors, associate professors and full professors) can enforce the promotion of a faculty member from assistant professor to associate professor, meanwhile, the same majority might be unable to enforce a promotion from associate professor to full professor. Indeed, this enforcement requires the approval of a majority of members having the same grade as or higher grade. Thus, the decision power of a coalition on the alternative depends on that alternative.

We show in this paper that Shenoy's [15] positive result extends for more general committee games that need not be simple. Some other results give some sufficient condition for the non-emptiness of the one-core of a committee game. Also, the concept of a bargaining set first introduced by Aumann and Maschler [2] in the context of games with side payments was defined by Shenoy [15] with appropriate modifications for committee games. As for the existence/non-emptiness, we also show that the bargaining set of less than 5-player committee game is always nonempty.

The rest of the paper is organized as follows: Section 2 is devoted to the model and preliminaries. We recall the formal definition of a committee game and describe how the game is played. We recall solution concepts introduced by Shenoy [15]. In Section 3, we present our results including a sufficient condition for non-emptiness of the one-core and the bargaining set of any committee game of less than 5 players. Conclusion, which is Section 4 ends the paper.

2. The setting and preliminaries

The model

Throughout the paper, the set of players that is, the committee is denoted by $N = \{1, 2, \ldots, n\}$, the finite set of candidates or outcomes is A. It is assumed that A has at least three elements. Nonempty subsets of N are called coalitions and the set of all coalitions of N is 2^N, $|T|$ stands for the cardinality of any set T.

Any player has a preference relation on A which is a weak order (reflexive, complete and transitive relation). It is well known that any such relation can be represented by a utility function. If $\succeq_i$ denotes the preference of voter i and a and b are two outcomes, $a \succeq_i b$ means that according to player i, outcome a is at least as good as outcome b. $a \succ_i b$ means that i strictly prefers a to b and $a \sim_i b$ means that i is indifferent between a and b. A profile is a collection of individual preferences, $(\succeq) = (\succeq_i)_{i\in N}$. The rule by which the committee members arrive at a decision is called the characteristic function which is a mapping $v : 2^N \to \mathcal{P}(A)$, where $\mathcal{P}(A)$ designates the set of subsets of A. For any coalition S, $v(S)$ denotes the subset of outcomes that coalition S can realize if the decision is unanimous in S. This means that at any time, an outcome x becomes the final outcome of the game, whenever a coalition S such that $x \in v(S)$ asks for the adoption of x. It is assumed that v satisfies the following conditions:

$$\forall S_1, S_2 \in 2^N, \quad S_1 \subseteq S_2 \Rightarrow v(S_1) \subseteq v(S_2) \tag{C_1}$$

$$v(N) = A \tag{C_2}$$

$$\forall S_1, S_2 \in 2^N, \quad \begin{cases} S_1 \cap S_2 = \emptyset \\ v(S_1) \neq \emptyset \neq v(S_2) \end{cases} \Rightarrow \begin{cases} v(S_1) = v(S_2) \\ |v(S_1)| = 1. \end{cases} \tag{C_3}$$

Condition (C_1) is the well known monotonicity condition, (C_2) means that the whole committee members can enforce any alternative. Condition (C_3) ensures that the committee decision consists of at most one outcome.

The tuple $\Gamma = (N, A, v, (\succeq))$ is called an (ordinal) n-person committee game. Note that $(\succeq)$ could also be replaced with a utility vector $u = (u^i)$, where $u^i : A \to \mathbb{R}$ denotes the real-valued ordinal utility function of player i. In this case, utility is assumed to be nontransferable and interpersonal comparison of utilities has no meaning. The committee aims at choosing one option from the set A of outcomes and the members are considered to be situated in one room. As in Shenoy [15], we are primarily concerned with small committees that arrive at a decision after (possibly) lengthy deliberations. In this respect the model considered here differs fundamentally from the theory of elections where the decision makers (the voters) are numerous and spread out extensively. Before recalling the manner the game is organized, let us remark that the committee game model fits very well into the more general model of social environments. A social environment is described by a tuple $(N, Z, (\to_S)_{S\in 2^N}, (\succeq_i)_{i\in N})$ where N is the set of players, Z the set of outcomes, $\{\to_S\}$ are effectiveness relations defined on Z. The relation $\to_S$ represents what coalition S can do; $a \to_S b$ means that if a is status quo, S can make b the new status quo. It does not mean that S can enforce b no matter what anyone else does. After the move of S to b, another coalition T might move to c and so on and so forth. Meanwhile in a committee game, if $b \in v(S)$ then S can enforce b. Social environments have been considered in many works in the literature including Chwe [7], Xue [18,19], Suzuki and Muto [16], Béal et al. [4] and Momo and Tchantcho [12]. Note that committee games generalize the model of simple games. In this respect, a committee game $\Gamma = (N, A, v, (\succeq))$ is said to be simple if: $\forall S \in 2^N$, $v(S) = \emptyset$ or $v(S) = A$.

If $v(S) = \emptyset$ then S is a losing coalition and if $v(S) = A$, S is a winning coalition. In a simple committee game, a coalition is a minimal winning coalition if it is a winning coalition and if every