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Sensitivity analysis of markup equilibria in complementary markets

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ABSTRACT

We study the competitive structure of a market in which firms compete to provide various products within a bundle. Firms adopt price functions proportional to their per-unit costs by selecting markups. We consider two measures reflecting, respectively, the intensity of direct competition and the impact of complementarity on each producer's markup. We characterize the sensitivity of these terms to various changes in the market structure and relate this to changes in producer profits and the social efficiency of the market.

1. Introduction

We consider markets where products are encoded by links in a series–parallel (SP) network. Customers purchase product bundles, given by paths of the network, where parallel links represent substitutes, while series links represent complements. The resulting market conditions are captured by the model of markup equilibrium discussed by Correa et al. [5] (hereafter CFLS). In this paper, we extend their analysis to study the sensitivity of prices, profits and welfare to changes in market conditions and competitive structure. This enables comparisons of competition across a broad set of alternative market configurations in which both complements and substitutes are present.

The CFLS model considers producers that face linear marginal costs and compete to provide all or some portion of the bundle to customers, who in turn choose a set of producers offering the lowest combined price (path in the network). The model relies on a form of supply function equilibria [9], in which producers set prices by choosing a markup to apply to production costs. This is an attractive modeling choice in the bundled setting, as scheduled quantity-dependent price adjustments remove the ambiguity around revenue-splitting that would result in a Cournot-type model of complementary producers. Supply function models yield a structure where bundle-level purchase quantities (i.e., path flows) uniquely determine both the producer-level purchase quantities (link flows) and the market price of each producer's output. In contrast, a pure quantity-commitment model lacks a mechanism for setting individual prices.

Practically, bundling is important to many industries. In freight shipping, point to point routes often involve multiple carriers, each servicing a distinct geography and/or mode of transport. The model also applies to decentralized assembly supply chains, where a manufacturer contracts separately to purchase components from various suppliers. Such outsourcing typically requires a modular product structure that is amenable to an SP representation. Taking the assembler as a monopsonistic buyer, one could employ our model to understand the market around individual component suppliers (e.g., producers of processors, hard disks, and displays in a computer system supply chain). The SP structure provides a general framework to study markets where customers have a need of a set of elements that compose the final product. The study of markets arising from a network structure has drawn attention from the business strategy community [17,3] and in the operations literature surrounding transportation networks [10,12,15,16], telecommunication and computing services [1,4,13] and decentralized assembly supply chains [7,14,8,11].

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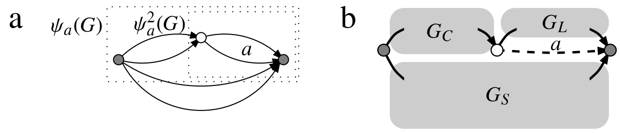

Fig. 1. (a) A series–parallel market with 6 producers. Boxes represent submarkets $\psi_a(G)$ and $\psi_a^2(G)$. (b) Competition for producer a at depth 3: SP networks G_C, G_S, and G_L are the complement, local and substitute markets of a, respectively.

We use the equilibrium characterization provided by [5] to derive a number of important structural insights within this framework. Among our findings are that:

(i) an increase in any producer's cost of production increases the markups of *all competitors* in equilibrium.
(ii) an increase in a producer's own costs *can increase* that producer's equilibrium profits.
(iii) an increase in the costs of production for complementary items *decreases bundle share* for efficient producers, but their less-efficient competitors may actually *gain bundle share*.
(iv) mergers that consolidate market power locally may in fact *improve social efficiency* in the full market for bundles.

At an intuitive level, the relationships we observe depend on whether certain producers interact more as competitors or as complementors. As we will show, both elements are present in most inter-producer relationships.

2. The SP markup equilibrium model of competition

This section provides a quick overview of the markup equilibrium model, as defined in CFLS; we refer the reader to [5] for details. This model is a special form of supply function equilibrium [9] where producers specify price functions by committing to a fixed percentage markup over per-unit production costs. The market is encoded by an SP network $G = (V_G, A_G)$, where each link $a \in A_G$ represents a producer. SP graphs are created by sequentially joining smaller SP graphs in series or in parallel, and capture well the modularity of constructing bundles, which are given by origin-to-destination paths. Indeed, the SP framework accommodates complements, substitutes, and multiple layers thereof. Let $\mathcal{B} := \{B_1 \dots B_m\}$ represent a set of bundles, all equivalent in the eyes of our customers. We say that $a \in B_i$ if producer a contributes to B_i. Producer a may contribute to multiple bundles so that its total production x_a equals $\sum_{B_i \ni a} f_i$ where the vector $f \in \mathbb{R}^m$ describes the allocation of consumption across bundles. A basic example is a computer system, with the option to purchase CPU, keyboard and monitor individually or in integrated bundles. A structure of this type is shown in Fig. 1(a).

Each producer faces a marginally-increasing cost curve and commits to an upward-sloping price function. For the analysis below, we assume that costs are linear and equal to $c_a x_a$ per unit. This leads to the price function $p_a(x_a) = \alpha_a c_a x_a$ per unit for the firm's chosen markup of $\alpha_a \geq 1$. Demand for otherwise-identical products will split into proportions that equalize price among active producers. Complementarity arises from the decentralized production of component products within some demanded bundle of goods.

Notation is needed to describe the recursive structure of SP networks. A *submarket* g refers to an SP subnetwork nested within G. We denote the join of a collection $\mathcal{g}$ of submarkets using the operators $P(\mathcal{g})$ and $S(\mathcal{g})$, respectively. Inversely, the mapping $\psi(g)$ returns the set of submarkets comprising g. A submarket is labeled either *series* or *parallel*, as indicated by the type of join applied last in its construction. We require when g is a series submarket that all elements of $\psi(g)$ be parallel submarkets, and vice versa, so that $\psi(g)$ represents the largest (by cardinality) set of submarkets from which g can be formed in a single composition. For submarkets $g' \subseteq g$, the restricted mapping $\psi_{g'}(g)$ selects the submarket of g that contains g'. We let $\nu_g := (G, \psi_g(G), \psi_g^2(G), \dots, \psi_g^{h_g}(G) = g)$ denote the unique sequence of submarkets starting with G within which g is nested, where h_g is the depth at which g is nested. For example, the sequence of submarkets ν_a in Fig. 1(a) is $(G, \psi_a(G), \psi_a^2(G), a)$. Finally, let $\nu_{g,P} = (g_1, g_2, \dots)$ (alternatively, $\nu_{g,S}$) be the subsequence of odd or even elements of ν_g obtained when restricting to only parallel (series) submarkets. The sequence $\nu_{g,P}$ provides the increasingly specific decisions that a customer must make before purchasing from g. Lastly, given $g \subseteq g'$, we use $g' \setminus g$ to denote the market in g' with producers from g removed and $\vec{\alpha}_{-g}$ to denote the markups vector of producers in $G \setminus g$.

The game has two phases: all producers choose markups simultaneously, followed by the allocation of an inelastic unit demand across the bundles. CFLS show that a unique markup equilibrium $\vec{\alpha}$, a unique production vector $\vec{x}$, and an aggregate bundle price p_G exist if and only if the network G is 3-edge-connected. We henceforth assume that G satisfies this property.

Several results discussed in CFLS will be useful as preliminaries. Fixing a markup vector $\vec{\alpha}$, one can construct a *price multiplier* $R_g(\vec{\alpha})$, used to compute the resulting price p_g as $d_g R_g(\vec{\alpha})$, where d_g is the demand for g. Price multipliers are constructed recursively according to $R_{S(\mathcal{g})}(\vec{\alpha}) = \sum_{g \in \mathcal{g}} R_g(\vec{\alpha})$ for a series market, $R_{P(\mathcal{g})}(\vec{\alpha}) = (\sum_{g \in \mathcal{g}} 1/R_g(\vec{\alpha}))^{-1}$ for a parallel submarket, and $R_a(\alpha_a) = \alpha_a c_a$ for a producer. Furthermore, the network can be pivoted around any submarket g to produce a *substitute network* $G \ominus g$ that encodes the local view of competition from g. When the full market is clear from the context we will omit it for brevity and just write $\ominus g$. The optimal markups for producers in g depend on producers outside g only through the aggregate multiplier $R_{\ominus g}$. Using this, a best-response markup of producer a to its competitors' markups $\vec{\alpha}_{-a}$ is $2 + R_{\ominus a}(\vec{\alpha}_{-a})/c_a$. This formula provides a system of equations that is used by CFLS to characterize equilibria.

While pivot $\ominus a$ redefines the network so that all paths act as substitutes for a, an additional scaling factor is needed to adjust the size of the relevant market to reflect the nature of complementarity introduced by producer a's competitors. The demand of a turns out to be $x_a = \mu_a R_{\ominus a}/(R_{\ominus a} + \alpha_a c_a)$, where the scaling factor is given by $\mu_a := \prod_{g \in \nu_{a,S}} R_{\ominus g}/\left(R_{\ominus g} + R_{g \setminus \psi_a(g)}\right)$.

3. Sensitivity analysis of producer outcomes

We now study the effects of changing market parameters on the outcome experienced by a producer a in equilibrium. The impact of any perturbation manifests itself as a combination of its effects on the multipliers $R_{\ominus a}$ and μ_a. We analyze these effects for a perturbation of a producer's own efficiency parameter, as well as for changes in the structure of its competition. In the latter case, we distinguish between those competitors whose markups decrease μ_a (net complements of a) and those whose markups increase μ_a (net substitutes of a).

3.1. Sensitivity of markups

In this section we formalize the impact of a fixed submarket g_F on the competition in another submarket g. We make the distinction between the full game and a *local game* on $\ominus g_F$. In the latter case, markups for producers in g_F are fixed – and aggregated together by R_{g_F} – while the game is played only among producers outside it. Hence, the demand becomes elastic: a small R_{g_F} indicates the existence of attractive options inside g_F. As R_{g_F} shrinks, the competition in $\ominus g_F$ becomes more intense. We let the *submarket response function* $\phi_{g|g_F}(R_{g_F})$ capture the value corresponding to R_g that arises from the equilibrium of the local