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On sabotage games

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ABSTRACT

Sabotage games on a graph involve Runner who wants to travel between two given vertices and Blocker who aims to prevent Runner from arriving at his destination by destroying edges. This paper completely characterizes games with multiple destinations on weighted trees for both local and global cutting rules of arbitrary capacity. The games on weighted graphs are characterized using the tree unraveling of the graph.

1. Introduction

On October 29, 2011 New York City has experienced an unusually early and heavy snowstorm. The storm left more than three million people without power and shut down critical landing systems at New York's major airports, disrupting air travel and forcing more than 150 flights to divert.

Among them was the JetBlue Flight 504 from Fort Lauderdale. Flight 504's pilot has asked to land in Boston. But Boston was soon swamped with two dozen diverted planes, some parked directly on the runways, closing it other landings. The air transportation network literally started to crumble under Flight 504's wings. It was then re-directed to Hartford, but it was also directly in the snowstorm's path. Flight 504 has been able to land but the airport's gates were already overcrowded, weather has worsened, and power outages were constant. After the three-hour flight 129 passengers and 6 crew members of Flight 504 has been forced to spend additional seven and a half hours on a tarmac without water and food. The outline of the Flight 504 incident follows Gerchick [3].

The travails of Flight 504 epitomize a pervasive problem: traversing a network while it is falling apart. The dual problem of restricting or blocking undesirable traffic in a given network is also widespread. For example, The Mexico–United States border is one of the world's largest drug smuggling corridors. The assets employed by the United States Border Patrol to interdict illegal entries can be broadly divided into two categories: the detection assets and the interception assets. The detection assets are sensors (small seismic or magnetic transmitters that are capable of detecting movement on the ground and transmitting the information to the nearest border patrol station) and heli-patrols (helicopters flying at low altitudes). The interception is performed by road and off-road patrols.

The set of all possible infiltration routes through the given border area – determined by the existing roads, paths, and physical characteristics of the terrain – constitutes the network. The infiltrator's objective is to avoid getting caught while traversing the network. The objective of the interdictor is to prevent the infiltration. This border security problem is inherently a dynamic process. The infiltrator who was detected once but avoided the interception can be detected, later on, by a different detection asset and another attempt at capture can be made in the different part of the network. Most of the interdictor's assets are transparent, their locations are perfectly observed by the infiltrator.

This paper aims to provide a game-theoretic framework suitable for the analysis of the general issue of dynamic network reliability or vulnerability. The scope for potential applications of the developed framework is very large. It can be used to study a wide variety of issues, including design of reliable networks for transportation, information exchange, computation, etc., as well as the dual problem of dismantling unwanted networks, or preventing the propagation of undesirable traffic of any kind in a given network. More specifically, the framework can be used to study, in a dynamic setting, positioning of the traffic control units; interception of drug traffic or smuggling; or routing of information in presence of the malicious opponent who wants to stop the communication.

The problem of network traversing in presence of adversary is modeled as a sequential two-player game. One player (Runner) wants to travel along the edges of the given graph from a starting

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node to a destination node. The other player (Blocker) tries to prevent Runner from arriving at the destination by blocking (destroying) edges in the graph. If, as in example of Flight 504, the role of Blocker is played by Nature then solving the game provides the characterization of the worst-case scenario. Following van Benthem [11], such games are called sabotage games.

The graph (or multigraph) on which the game is played is referred to as the sabotage graph. In case of multigraph an edge is characterized by an integer capacity (or weight) that is interpreted as a number of different direct routes between two vertices. The set of actions available to Blocker is referred to as cutting rules. It is important to distinguish between the "local" and the "global" cutting rules. We say that Blocker obeys the local cutting rule if the edges Blocker is able to delete at her move are only those incident with the current vertex of Runner. In contrast, the global cutting rule allows Blocker to delete an edge anywhere in the graph, independent of the current position of Runner. Similarly, the set of actions available to Runner as the game evolves is referred to as moving rules. The "local" Runner can only travel from his current vertex to an adjacent one. The global movement rule would allow Runner to "secure" an edge anywhere in the graph with the provision that once secured, the edge cannot be destroyed by Blocker anymore. Runner wins if he is able to secure a path from the origin vertex to the destination vertex. Blocker is also characterized by a "capacity of a cut", that is the total capacity of the edges she is allowed to cut at any of her moves.

2. Related literature

The problem of dynamic graph reliability was first introduced by Papadimitriou [8] as a problem of decision-making under uncertainty. He considers a player who wishes to traverse the graph while it is falling apart and the probability of an edge failure before the next move of the player depends on his current position. Sabotage games have been introduced by van Benthem [11] and extensively studied from the perspective of computer science [6,10,5]. The closest work is Radmacher and Thomas [9] which also points out the difference between local and global rules and uses game-theoretic terminology.

Another related strand of the literature is literature on network interdiction (see Collado and Papp [2]). Network interdiction is the monitoring or halting of an adversary's activity on a network. The evader operates on the network to optimize some objective such as moving through the network as fast as possible, or with as little probability of being detected as possible, or to maximize the amount of goods transported through the network. The interdictor has the ability to change the structure or parameters of the network in order to minimize the evader's objective function. Starting with Washburn and Wood [12] this literature often takes the game-theoretic approach, using two-person simultaneous move zero-sum games. Hong [4] studies a strategic network flow interdiction problem. The evader chooses a flow specifying a plan for carrying bads through a network from a base to a target. Simultaneously, the interdictor chooses a blockage specifying a plan for blocking the flow of bads through arcs in the network.

Games of pursuit and evasion, the angel and the square-eater game (Berlekamp et al. [1]) and the cops and robbers game (Nowakowski and Winkler [7]), can be viewed as very specific generalization of sabotage games but their solutions rely on a very different techniques.

3. Definitions

3.1. Weighted graphs

A graph $G = (V, E)$ is a finite non-empty set V of *vertices* and a set E of edges that connect some of the vertices. An *edge* is an unordered pair of distinct vertices. Two vertices $v_\imath$ and $v_\jmath$ are *adjacent* if $v_\imath v_\jmath$ is an edge of G; the vertices $v_\imath$ and $v_\jmath$ are *incident* with the edge $v_\imath v_\jmath$. A (simple) *path* on the graph G is a sequence of distinct vertices (except possibly, the first and the last), $v_1, v_2, \ldots, v_n$ such that any two consecutive vertices in the sequence are adjacent. A *cycle* is a path with more than three vertices in which the first and the last vertices are the same. A graph is *connected* if there is a path between each pair of its vertices.

To describe sabotage networks with multiple edges the notion of a weighted graph is used. A *weighted graph* $G = (V, E, w)$ is a finite non-empty set of vertices V, a set of edges E, and a weight function w that associates a non-negative integer – weight or capacity – with every edge of the graph. Two vertices $v_\imath$ and $v_\jmath$ are adjacent if the weight $w(v_\imath v_\jmath) > 0$. The values of the weight function are interpreted as the number of edges between vertices. The definitions of a path, a cycle, and connectivity for weighted graphs are the same as for graphs.

A *weighted tree* is a connected weighted graph with no cycles. As a result, there is a unique path between any two vertices of the given tree. A *rooted tree* has a distinguished vertex called the root, that induces a natural partial order on vertices, away from (or towards) the root. In a rooted tree, the *parent* of a vertex is the vertex adjacent to it on the path to the root; every vertex except the root has the unique parent. A *child* of a vertex v is a vertex of which v is the parent. A *terminal vertex* (or a leaf) is a vertex that has no children.

3.2. The game

A two-player sabotage game is played on a given connected weighted graph $G(V, E, w)$, referred to as the sabotage graph. Player 2, called Runner, is initially located at the *starting vertex* v_0 and wants, by traversing the graph to arrive in his *set of destinations*, D. The set of destinations may be a singleton or it may contain more than one vertex. In the latter case, Runner wins by arriving at either vertex in D. The objective of Player 1, called Blocker, is to prevent Runner's arrival at a destination by deleting edges in the graph. Thus, Blocker wins whenever Runner is not in the destination set and there exists no path from his current position to any of the destination vertices.

Players move sequentially starting with Blocker. Blocker is endowed with the strictly positive integer *cut capacity*, c, which is the maximal total weight of the edges she can delete at any one move. In addition, Blocker is subject to *cutting rules*. Under the local cutting rule, Blocker is only able to delete edges incident to the current position of Runner subject to not exceeding her cut capacity. Under the global cutting rule, Blocker is able to reduce the weight of any number of edges anywhere on the graph but again subject to not exceeding her cut capacity. For example, a global Blocker with cut capacity of 2, can reduce the weight of any edge by two, reduce the weight of two edges by one, reduce the weight of just one edge by one, or do nothing.

When it is Runner's turn to move, Runner can travel from his current vertex $v_\imath$ to any vertex $v_\jmath$ provided that $w(v_\imath, v_\jmath) > 0$. That is Runner can travel to any vertex adjacent to the one he is currently at. After the move, Runner's current position on the graph becomes $v_\jmath$. All the results in the paper are derived for the case of the local Runner.

Thus, every move of Runner changes his position on the graph and every move of Blocker changes the weight function of the graph. A position in the sabotage game then is a pair (v_i, w_j), where v_i is the current vertex of Runner and w_j is the current weight function of the graph. The set of all legal positions (positions reached from the initial position by a sequence of feasible moves of Blocker and Runner) is denoted by $\mathcal{P}$. A play of the game is