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More on batched bin packing

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ABSTRACT

Bin packing is the problem of partitioning a set of items into subsets of total sizes at most 1. In batched bin packing, items are presented in k batches, such that the items of a batch are presented as a set, to be packed before the next batch. In the disjunctive model, an algorithm must use separate bins for the different batches. We analyze the asymptotic and absolute approximation ratios for this last model completely, and show tight bounds as a function of k.

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1. Introduction

We study the bin packing problem [23,15], and analyze algorithms for it that receive the input in a small number of batches. In the bin packing problem, the goal is to allocate input items of sizes in $(0, 1]$ to blocks called bins, such that the total size of items assigned to each block does not exceed 1, and the number of non-empty bins is minimized. The process of allocation of items to bins is also referred to as the process of packing items into bins. The items are denoted by $1, 2, \dots, n$, and the size of item i is denoted by s_i.

In the offline variant, all input items are presented together as a set. The problem is NP-hard in the strong sense, and thus, approximation algorithms were studied. An approximation algorithm has an asymptotic approximation ratio of at most R, if there exists a constant $C_1 \geq 0$ (which is independent of the input), such that for any input I, the cost of the algorithm for I does not exceed the following value: R times the optimal cost for this input plus C_1. If $C_1 = 0$, then the approximation ratio is called absolute. A specific optimal algorithm as well as its cost are denoted by $OPT(I)$ or OPT, when the input is fixed. An alternative definition of the asymptotic approximation ratio is the supreme limit of the ratio between the cost of the algorithm and OPT, as a function of this last cost, taking the maximum or supremum over the inputs with the same optimal cost.

For this classic variant, an asymptotic fully polynomial time approximation scheme (AFPTAS) is known [11,18]. This is a class of algorithms containing an approximation algorithm with an asymptotic approximation ratio of $1 + \varepsilon$ for any $\varepsilon > 0$, with running time polynomial in the size of the input and in $\frac{1}{\varepsilon}$. Many fast heuristics are known [23,17,16], including First-Fit (FF) [23,17], which processes a list of items, and assigns each item, in turn, to the minimum index bin where it can be added. First-Fit-Decreasing (FFD) [15] acts identically to FF, but it requires the list of input items to be sorted according to non-increasing sizes. FFD is known to have the best possible absolute approximation ratio 1.5 (this is the best possible unless $P = NP$) [22]. Next-Fit (NF) [16] assigns each item into the maximum index non-empty bin if it can be packed there, and otherwise to the minimum index empty bin. Next-Fit-Decreasing (NFD) [1] and Next-Fit-Increasing (NFI) [12] are the algorithms that apply NF to lists that are sorted by non-increasing and non-decreasing orders, respectively. First-Fit-Increasing (FFI) is identical to NFI. The asymptotic and absolute approximation ratio of FF and NF are 1.7 and 2, respectively [17,8,16]. The sorted versions have a better performance, FFD has an asymptotic approximation ratio of 11/9 [15], while NFD and NFI have an asymptotic approximation ratio of approximately 1.69103 [1,12].

In the online scenario, an algorithm is presented with the items one by one, and each item must be packed before the next item can be seen. In this variant, the best possible asymptotic approximation ratio (also called competitive ratio, for online algorithms, as an online algorithm is compared to an optimal offline algorithm) is at least 1.5403 [4] and at most 1.58889 [21], and the best possible absolute approximation ratio is $\frac{5}{3}$ [27,3]. The algorithms FF and NF are online algorithms, but FFD, NFD, and NFI, are not online algorithms.

In batched bin packing, items are presented in k batches, for an integer $k \geq 1$. For each batch, the algorithm receives all its items at once, and these items are to be packed irrevocably before the

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next batch is presented (if the current batch is not the last one). This last model is an intermediate model, which bridges between the two extreme known models. The case $k = 1$ corresponds to the offline problem. If the number of batches that may be presented is unbounded, this scenario corresponds to the online problem. There are two models for any fixed $k \geq 2$. In the disjunctive model [7], the algorithm must use separate bins for the different batches. In the augmenting model [14], the algorithm may use existing bins, where items were already packed in previous batches, as well as new bins. Obviously, any algorithm for the disjunctive model can be seen as an algorithm for the augmenting model, with the same performance.

In this work, we analyze the asymptotic approximation ratio and the absolute asymptotic approximation ratio for the disjunctive model completely, and show tight bounds for them as a function of k. The asymptotic approximation ratio tends to approximately 1.69103, while the tight absolute approximation ratio is exactly k. Moreover, our results provide an improved upper bound on the asymptotic approximation ratio for the augmenting model with two batches. This last algorithm has an asymptotic approximation ratio of 1.5, improving over the algorithm of Dósa [7] (see below). For the analysis, we will define subset of items called *combined items*, and use them for the analysis of an optimal solution rather than dealing with the actual items. Moreover, we analyze a particular offline algorithm for the combined items, rather than analyzing an optimal solution. The other features of our analysis are related to those used in [1,19,25,13,10,7]. Finally, we show that the absolute approximation ratio for the augmenting model with two batches is $\frac{3}{2}$, while for at least three batches, it is exactly $\frac{5}{3}$.

There is an additional relation between batched bin packing and the online problem. All lower bounds on the asymptotic competitive ratio [26,24,4] are of the form where a pre-determined number batches of identical items are presented to the algorithm, such that the algorithm does not know how many non-empty batches will arrive. These lower bounds are valid for both models of batched bin packing, with the corresponding number of batches (the number of batches in the lower bound construction). Moreover, there is a certain relation of the disjunctive model to bounded space online bin packing. In the latter problem, an online algorithm may keep a constant number of bins open, and it must close all other bins that were used, in the sense that they can no longer be used for packing new items. The lower bounds for online bounded space bin packing are of the form where batches of identical items arrive, and the algorithm must pack almost all items (except for a constant number of items) of a batch into new bins, as there is only a constant number of open bins that were used before. The best possible asymptotic competitive ratio for online bounded space bin packing is the sum of a series and tends to approximately 1.69103 [19,25]. This is the same value as the asymptotic approximation of NFD and NFI, and the series will be discussed in what follows.

In [14], Gutin, Jensen, and Yeo, proved a lower bound of approximately 1.3871 on the asymptotic approximation ratio for the augmenting model with two batches. In [7], Dósa analyzed FFD for two batches and both models of batched bin packing. The algorithm applies FFD on each batch independently, using separate bins. The asymptotic approximation ratio was shown to be $\frac{19}{12} \approx 1.5833$ even for the disjunctive model. Moreover, in the same paper it is shown that the asymptotic approximation ratio of any algorithm for the disjunctive model (and $k = 2$) is at least $\frac{3}{2}$. In the case $k \geq 3$, there are online algorithms that perform better than the best algorithms for the disjunctive model (which are analyzed here), and thus better algorithms for the augmenting model cannot be those of the disjunctive model. Balogh et al. [2] proved lower bounds on the asymptotic competitive ratios of algorithms for the augmenting model and different numbers of batches, and in particular, they showed a lower bound of 1.51211 for three batches (while the previously known lower bound was 1.5 [26]). For four batches, the lower bound of van Vliet [24] is approximately 1.539, thus, the effect of a small number of batches is mostly noticeable for $k = 2$ and $k = 3$.

2. Main result

We will prove our results in this section. We start with the required definitions, then we prove the upper bounds, and finally we will show that these bounds cannot be improved for the disjunctive model.

Preliminaries. We define a sequence π_j (for any integer $j \geq 1$) as follows. Let $\pi_1 = 1$, and for $i \geq 1$, $\pi_{i+1} = \pi_i(\pi_i + 1)$. Let $\Gamma_k = \sum_{i=1}^{k} \pi_i$. We have $\Gamma_1 = 1$, $\Gamma_2 = \frac{3}{2}$, $\Gamma_3 = \frac{5}{3}$, $\Gamma_4 = \frac{71}{42} \approx 1.690476$, and $\Gamma_\infty = \lim_{k\to\infty} \Gamma_k \approx 1.69103$. Recall that the last value is the asymptotic approximation ratio of several well-known bin packing algorithms (for example, it is the asymptotic approximation ratio of Harmonic, NFI, and NFD). This sequence is frequently used in analysis of bin packing [1,19,25,13,10]. In particular, the following claim is often used, and we will use it in our analysis as well.

Claim 1. *For any integer $i \geq 2$, $\pi_i > \pi_{i-1}$, and $\pi_i \geq i$. For any two integers $i \geq 1$ and $i' \geq i$, $\pi_{i'}$ is divisible by π_i. Moreover, we have $\sum_{i=1}^{j} \frac{1}{\pi_i+1} = 1 - \frac{1}{\pi_{j+1}}$ for $j \geq 1$.*

Proof. The claims are proved by induction. In the base case $i = 2$, $\pi_2 = 2$. Assume that $\pi_i > \pi_{i-1}$ and $\pi_i \geq i$ hold. Then, $\pi_{i+1} = \pi_i(\pi_i + 1)$. Using $\pi_i \geq i$, we get $\pi_i + 1 > i \geq 2$, so $\pi_{i+1} > \pi_i$. Using $\pi_i \geq i \geq 2$, we get $\pi_i(\pi_i + 1) \geq 2(i + 1) > i + 1$.

The base case of the second claim ($i' = i$) is trivial, and the induction is simple as well: Assume that $\pi_{i'-1}$ is divisible by π_i. This implies that $\pi_{i'}$ is divisible by π_i as $\pi_{i'}$ is divisible by $\pi_{i'-1}$.

Consider the third claim. In the base case $j = 1$, and $\frac{1}{\pi_1+1} = \frac{1}{2} = 1 - \frac{1}{2} = 1 - \frac{1}{\pi_2}$. Next, assume that $\sum_{i=1}^{j} \frac{1}{\pi_i+1} = 1 - \frac{1}{\pi_{j+1}}$ holds. We show that $\sum_{i=1}^{j+1} \frac{1}{\pi_i+1} = 1 - \frac{1}{\pi_{j+2}}$ holds as well. Indeed, $\sum_{i=1}^{j+1} \frac{1}{\pi_i+1} = \sum_{i=1}^{j} \frac{1}{\pi_i+1} + \frac{1}{\pi_{j+1}+1} = 1 - \frac{1}{\pi_{j+1}} + \frac{1}{\pi_{j+1}+1} = 1 - \frac{1}{\pi_{j+1}(\pi_{j+1}+1)} = 1 - \frac{1}{\pi_{j+2}}$. ■

In what follows, we will prove the following theorem.

Theorem 2. *The absolute approximation ratio for batched bin packing in the disjunctive model is k, and the asymptotic approximation ratio is Γ_k. The additive constant for the last approximation ratio is $\Theta(k)$, and for $k = 2$ it is exactly $\frac{1}{2}$.*

Upper bounds. We will analyze a specific input I. Let OPT_i denote the optimal cost for packing the items of batch i, let ALG_i denote the cost of a given algorithm ALG for batch i, and $ALG = \sum_{i=1}^{k} ALG_i$. Obviously, $OPT_i \leq OPT$.

Consider the following algorithm. The algorithm FF-Batch (FFB) applies FF on each batch separately.

Proposition 3. *For any $k \geq 2$, the absolute approximation ratio of FFB is at most k.*

Proof. If the input is empty, we are done. If FFB uses a single bin for every batch, $FFB_i = 1$, and $FFB = k$. In the latter case, as $OPT \geq 1$, the absolute approximation ratio does not exceed k.

Finally, assume that at least two bins were used by FFB for at least one batch. Let θ be such that $1 \leq \theta \leq k$ is the number of batches for which there are at least two bins, and let ℓ denote the total number of bins in such batches. For inputs where FF creates at least two bins, the total size of items packed into each bin is above $\frac{1}{2}$ on average (as any item of a bin could not be packed into a bin

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