Journal of the Korean Statistical Society 45 (2016) 314–328

Contents lists available at ScienceDirect

Journal of the Korean Statistical Society

journal homepage: www.elsevier.com/locate/jkss

Estimation and testing procedures for the reliability functions of generalized half logistic distribution

Ajit Chaturvedi [a], Suk-Bok Kang [b,*], Anupam Pathak [a]

[a] Department of Statistics, University of Delhi, Delhi-110007, India

[b] Department of Statistics, Yeungnam University, 280 Daehak-ro, Gyeongsan 38541, Republic of Korea

ARTICLE INFO

Article history:
Received 6 October 2014
Accepted 25 November 2015
Available online 19 December 2015

AMS 2000 subject classifications:
62F10
62F03

Keywords:
Generalized half logistic distribution (GHLD)
Point estimation
Sampling scheme of Bartholomew (1963)
Testing procedures
Type II cesoring

ABSTRACT

Two measures of reliability are considered, $R(t) = P(X > t)$ and $P = P(X > Y)$. Estimation and testing procedures are developed for $R(t)$ and P under Type II cesoring and a sampling scheme of Bartholomew (1963). Two types of point estimators are considered (i) uniformly minimum variance unbiased estimators (UMVUEs) and (ii) maximum likelihood estimators (MLEs). A new technique of obtaining these estimators is introduced. A comparative study of different methods of estimation is done. Testing procedures are developed for the hypotheses related to different parametric functions.

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1. Introduction and preliminaries

The reliability function $R(t)$ is defined as the probability of failure-free operation until time t. Thus, if the random variable (rv) X denotes the lifetime of an item or system, then $R(t) = P(X > t)$. Another measure of reliability under stress–strength set-up is the probability $P = P(X > Y)$, which represents the reliability of an item or system of random strength X subject to random stress Y. A lot of work has been done in the literature for the point and interval estimation and testing for $R(t)$ and P under censorings and complete sample case. For a brief review, one may refer to Bartholomew (1957, 1963), Basu (1964), Chao (1982), Chaturvedi and Pathak (2012, 2013, 2014), Chaturvedi and Rani (1997, 1998), Chaturvedi and Singh (2006, 2008), Chaturvedi and Surinder (1999), Chaturvedi and Tomer (2002, 2003), Constantine, Karson, and Tse (1986), Johnson (1975), Kelley, Kelley, and Schucany (1976), Pugh (1963), Sathe and Shah (1981), Tong (1974, 1975), Tyagi and Bhattacharya (1989), and others.

Half logistic model, obtained as the distribution of the absolute standard logistic variate, is probability model considered by Balakrishnan (1985). Balakrsihnan and Hossain (2007) considered generalized (Type II) version of logistic distribution and derived some interesting properties of the distribution. Ramakrsihnan (2008) considered two generalized versions of HLD namely Type I and Type II along with point estimation of scale parameters and estimation of stress–strength reliability based on complete sample. Arora, Bhimani, and Patel (2010) obtained the MLE of the shape parameter in a GHLD based on Type I progressive censoring with varying failure rates. Kim, Kang, and Seo (2011) proposed Bayes estimators of the

* Corresponding author. Tel.: +82 53 810 2323.
E-mail address: sbkang@yu.ac.kr (S.-B. Kang).

http://dx.doi.org/10.1016/j.jkss.2015.11.007
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shape parameter and the reliability function for the GHLD based on progressively Type II censored data under various loss functions. Seo, Lee, and Kang (2012) developed an entropy estimation method for upper record values from the GHLD. Azimi (2013) derived the Bayes estimators of the shape parameter and the reliability function for the GHLD based on Type II doubly censored samples. Seo and Kang (2014) derived the entropy of a GHLD by using Bayes estimators of an unknown parameter in the GHLD based on Type II censored samples. They also compared these estimators in terms of the mean squared error and the bias.

Let the life X of an item have the GHLD, then cumulative distribution function (cdf) and probability density function (pdf) of the (rv) X are, respectively

$$F(x;\lambda) = 1 - \left(\frac{2e^{-x}}{1+e^{-x}}\right)^{\lambda}, \quad x > 0,\ \lambda > 0$$

and

$$f(x;\lambda) = \frac{\lambda}{1+e^{-x}}\left(\frac{2e^{-x}}{1+e^{-x}}\right)^{\lambda}, \quad x > 0,\ \lambda > 0. \tag{1.1}$$

Here, it should be noted that λ is the shape parameter and, for $\lambda = 1$, it comes out to be the half-logistic distribution.

The purpose of the present paper is many-fold. We develop point estimation procedures under Type II censoring and a sampling scheme proposed by Bartholomew (1963). Testing procedures are also proposed. As far as point estimation is considered, we derive UMVUEs and MLEs. A new technique of obtaining UMVUEs and MLEs is developed. For obtaining UMVUEs, the major role is played by the estimators of the powers of the parameter. With the help of estimators of the powers of the parameter, we obtain the estimators of pdf at a specified point, which is subsequently used to obtain the UMVUEs of $R(t)$ and P. The MLEs of the parameter is derived. Utilizing the invariance property of the MLEs, the MLE of the pdf at a specified point is obtained, which is subsequently used to obtain the MLEs of $R(t)$ and P. Thus, we have established an interrelationship between various estimation problems and functional forms of the parametric functions to be estimated are not needed.

In Sections 2 and 3, respectively, we provide point estimators under Type II cesoring and a sampling scheme proposed by Bartholomew (1963). In Section 4, we developed test procedures. In Section 5, we present numerical findings. Finally, in Section 6, discussions are made and conclusions are presented.

2. Point estimators under Type II censoring

Suppose n items are put on a test and the test is terminated after the first r ordered observations are recorded. Let $0 < X_{(1)} \le X_{(2)} \cdots \le X_{(r)}, 0 < r < n$, be the lifetimes of first r ordered observations. Obviously, $(n-r)$ items survived until $X_{(r)}$.

Lemma 1. *Let $S_r = \sum_{i=1}^{r} \ln\left\{\frac{1}{2}\left(e^{x_{(i)}}+1\right)\right\} + (n-r)\ln\left\{\frac{1}{2}\left(e^{x_{(i)}}+1\right)\right\}$. Then, S_r is complete and sufficient for the distribution given at (1.1). Moreover, the pdf of S_r is*

$$g(s_r;\lambda) = \frac{\lambda^r s_r^{r-1}}{\Gamma(r)}\exp(-\lambda s_r), \quad s_r > 0. \tag{2.1}$$

Proof. (1.1) can be written as

$$f(x;\lambda) = \frac{\lambda}{1+e^{-x}}\exp\left(-\lambda\ln\left\{\frac{1}{2}\left(e^{x}+1\right)\right\}\right), \quad x > 0,\ \lambda > 0. \tag{2.2}$$

From (2.2), the joint pdf of $X_{(1)} \le X_{(2)} \le \cdots X_{(n)}$ is

$$f^*(x_{(1)}, x_{(2)}, \ldots, x_{(n)};\lambda) = n!\lambda^n \prod_{i=1}^{n}\left(\frac{1}{1+e^{-x_{(i)}}}\right)\exp\left(-\lambda\sum_{i=1}^{n}\ln\left\{\frac{1}{2}\left(e^{x_{(i)}}+1\right)\right\}\right). \tag{2.3}$$

Integrating out $x_{(r+1)}, x_{(r+2)}, \ldots, x_{(n)}$ from (2.3) over the region $x_{(r)} \le x_{(r+1)} \le \cdots \le x_{(n)}$, the joint pdf of $X_{(1)} \le X_{(2)} \le \cdots X_{(r)}$ comes out to be

$$h(x_{(1)}, x_{(2)}, \ldots, x_{(r)};\lambda) = \lambda^r n(n-1)\ldots(n-r+1)\prod_{i=1}^{r}\left(\frac{1}{1+e^{-x_{(1)}}}\right)\exp(-\lambda s_r). \tag{2.4}$$

It follows easily from (2.2) that the rv $U = \lambda\ln\left\{\frac{1}{2}\left(e^{x}+1\right)\right\}$ has exponential distribution with mean life $1/\lambda$. Moreover, if we consider the transformation $Z_i = (n-i+1)\{U_{(i)} - U_{(i-1)}\},\ i = 1, 2, \ldots, r;\ U_0 = 0$, then Z_i's are independent and identically distributed (i.i.d.) rv's, each having exponential distribution with mean life $1/\lambda$. It is easy to see that $\sum_{i=1}^{r} Z_i = S_r$. Result (2.1)

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