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# Semiparametric inference with a functional-form empirical likelihood

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ABSTRACT

A functional-form empirical likelihood method is proposed as an alternative method to the empirical likelihood method. The proposed method has the same asymptotic properties as the empirical likelihood method but has more flexibility in choosing the weight construction. Because it enjoys the likelihood-based interpretation, the profile likelihood ratio test can easily be constructed with a chi-square limiting distribution. Some computational details are also discussed, and results from finite-sample simulation studies are presented.

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## 1. Introduction

The empirical likelihood method, proposed by Owen (1988, 1990), provides a useful tool for obtaining nonparametric confidence regions for statistical functionals. Even though the empirical likelihood method is a nonparametric approach in the sense that it does not require a parametric model for the underlying distribution of the sample observation, the empirical likelihood method enjoys some of the desirable properties of the likelihood-based method. Using a nonparametric likelihood function, the empirical likelihood method can easily incorporate known constraints on parameters and also incorporate prior information in parameters obtained from other sources. For example, Chen and Qin (1993) and Qin (2000) discuss combining information using the empirical likelihood. A comprehensive overview of the empirical likelihood method is provided by Owen (2001).

We consider an extension of the empirical likelihood method by providing a class of nonparametric estimators that have the same asymptotic properties as the empirical likelihood method. In particular, instead of assuming a nonparametric likelihood, we consider a generalization of the empirical likelihood that uses a functional-form likelihood function in the likelihood maximization. The class of functional-form likelihood function contains the empirical likelihood function as a special case. The functional-form likelihood approach provides several useful alternatives to the classical empirical likelihood method in the sense that some of the computational difficulty of the empirical likelihood method can be avoided, and more clear insights can be obtained from the empirical likelihood method.

Let $z_1, \ldots, z_n$ be $n$ independent realizations of a vector-valued random variable $Z$ with a distribution function $F(z)$ that is completely unspecified. In the empirical likelihood approach, we consider a class of distribution functions, $\mathcal{F}_1 \subset \mathcal{F}$, where

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$\mathcal{F}$ denotes the general class of distribution functions, and $\mathcal{F}_1$ have support on $\{z_1, \ldots, z_n\}$. Thus, the elements in $\mathcal{F}_1$ can be written as

$$F_w(x) = \sum_{i=1}^{n} w_i I(z_i \leq x)$$

with $\sum_{i=1}^{n} w_i = 1$ and $w_i > 0$, where $I(z_i \leq x)$ takes the value one if $z_i \leq x$ and takes the value zero otherwise. The parameter $w_i$ is the amount of point mass that unit $z_i$ represents in the population. We are interested in making an inference about $\theta_0$ that is defined as a unique solution to $E\{U(Z;\theta)\} = 0$, where $U(Z;\theta)$ is an $r$-dimensional vector of some function known up to $\theta$ and the dimension of $\theta$ equals $p \leq r$. Hansen (1982) and Imbens (1997) considered this over-identified situation in the context of a generalized method of moments in econometrics.

In this setup, Qin and Lawless (1994) considered the empirical likelihood estimator of $\theta_0$ that can be obtained by maximizing

$$\sum_{i=1}^{n} \log(w_i) \tag{1}$$

subject to

$$\sum_{i=1}^{n} w_i \{1, U(z_i;\theta)\} = (1, 0). \tag{2}$$

Note that (2) is equal to the condition $E\{U(Z;\theta)\} = 0$ for $F \in \mathcal{F}_1$. Using the Lagrange multiplier method, the empirical likelihood estimator can be obtained by maximizing

$$l_e(\theta) = \sum_{i=1}^{n} \log\{w_i(\theta)\},$$

where $w_i(\theta)$ is of the form

$$w_i(\theta) = \frac{1}{n} \frac{1}{1 + \hat{\lambda}_\theta^T U(z_i;\theta)} \tag{3}$$

and $\hat{\lambda}_\theta$ satisfies the second equation of (2) by plugging (3) into (2). Qin and Lawless (1994) showed that the empirical likelihood estimator satisfies

$$2\left\{l_e(\hat{\theta}) - l_e(\theta_0)\right\} \to^d \chi_p^2 \tag{4}$$

where $\to^d$ denotes the convergence in distribution, $\chi_p^2$ denotes the chi-squared distribution with degree of freedom equal to $p$ and $\hat{\theta}$ is the empirical likelihood estimator. The result (4) is often called Wilk's theorem for empirical likelihood and is quite useful in obtaining confidence regions for $\theta_0$.

The weight (3) used to compute the empirical likelihood estimator can be expressed as

$$w_i(\theta, \hat{\lambda}_\theta) = \frac{m\left\{\hat{\lambda}_\theta^T U(z_i;\theta)\right\}}{\sum_{j=1}^{n} m\left\{\hat{\lambda}_\theta^T U(z_j;\theta)\right\}}, \tag{5}$$

where $m(x) = 1/(1+x)$ and $\hat{\lambda}_\theta = \hat{\lambda}(\theta; z_1, \ldots, z_n)$ satisfies

$$\sum_{i=1}^{n} w_i(\theta, \hat{\lambda}_\theta) U(z_i;\theta) = 0. \tag{6}$$

The Lagrange multiplier $\hat{\lambda}_\theta = \hat{\lambda}(\theta; z_1, \ldots, z_n)$ is completely determined by (6). We assume that, for given $\theta$, the solution $\hat{\lambda}_\theta$ to (6) is uniquely determined. The unique solution exists for any given $\theta$ if 0 is inside the convex hull of the points $U(z_1;\theta), \ldots, U(z_n;\theta)$.

We consider an extension of the empirical likelihood estimator by allowing that $m(x)$ in (5) can be some smooth function other than $m(x) = 1/(1+x)$. We call the proposed estimator a functional-form empirical likelihood (FEL) estimator since it involves a known function $m(x)$ in the probability weight. For example, the exponential tilting (ET) estimator considered in Kitamura and Stutzer (1997) and Schennach (2007) can be obtained by the same form (5) with $m(x) = \exp(x)$. Imbens, Spady, and Johnson (1998) advocated using the ET estimator over the empirical likelihood (EL) estimator based on Monte Carlo investigation and analytic comparison using higher order asymptotic expansion. In this paper, we discuss some asymptotic properties for the FEL estimator. In particular, asymptotic normality and a version of Wilk's theorem for the FEL

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