Statistical Methodology 31 (2016) 8–19

Contents lists available at ScienceDirect

Statistical Methodology

journal homepage: www.elsevier.com/locate/stamet

A new skew integer valued time series process

Marcelo Bourguignon [a,*], Klaus L.P. Vasconcellos [b]

[a] *Universidade Federal do Rio Grande do Norte, Departamento de Estatística, Lagoa Nova, 59078-970, Natal, RN, Brazil*

[b] *Universidade Federal de Pernambuco, Departamento de Estatística, Cidade Universitária, 50740-540, Recife, PE, Brazil*

ARTICLE INFO

Article history:
Received 11 January 2015
Received in revised form
4 May 2015
Accepted 9 January 2016
Available online 20 January 2016

Keywords:
INAR(1) process
Integer-valued time series
Thinning operator
Yule–Walker estimator

ABSTRACT

In this paper, we introduce a stationary first-order integer-valued autoregressive process with geometric–Poisson marginals. The new process allows negative values for the series. Several properties of the process are established. The unknown parameters of the model are estimated using the Yule–Walker method and the asymptotic properties of the estimator are considered. Some numerical results of the estimators are presented with a brief discussion. Possible application of the process is discussed through a real data example.

1. Introduction

Integer-valued times series with support in $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ are very frequent in practice. The need for modeling and analyzing counting data (negative and positive) occurs in many fields of real life such as medicine [11], image analysis [8], sports applications [12], financial applications [2] and so on.

Models for time series defined on the set $\mathbb{Z}$ of integers have been discussed by various researchers. Kim and Park [13] introduced an integer-valued autoregressive (INAR) process of order p with signed binomial thinning operator. Zhang et al. [23] introduced pth-order integer valued autoregressive processes with a signed generalized power series thinning operator. Kachour and Truquet [9] introduced a more general class, based on a modified version of the generalized thinning operator, also called the signed thinning operator. Kachour and Yao [10] proposed the first-order rounded integer-valued

* Corresponding author.
E-mail addresses: m.p.bourguignon@gmail.com (M. Bourguignon), klaus@de.ufpe.br (K.L.P. Vasconcellos).

http://dx.doi.org/10.1016/j.stamet.2016.01.002
1572-3127/

autoregressive process, based on the rounding operator. Some of these models arise as the difference between two discrete distributions. For example, Freeland [6] defined the true integer-valued autoregressive process of order one as the difference of two Poisson INAR(1) processes [1] which requires observing the two processes. Recently, Barreto-Souza and Bourguignon [4] introduced a stationary AR(1) process on $\mathbb{Z}$ with skew discrete Laplace marginals (which are distributed as a difference between two geometric random variables); this model is named skew INAR(1) process on $\mathbb{Z}$.

In a similar manner, we introduce a stationary first-order integer-valued autoregressive process with geometric–Poisson marginals (which are distributed as a difference between geometric and Poisson random variables), named new skew INAR(1) process (NSINAR(1)). Additionally, we will provide a comprehensive account of the mathematical properties of the proposed new process. The motivation for such a process arises from its potential in modeling and analyzing integer valued time series for which the non-negative part presents greater overdispersion than the negative part. The price change data can satisfying this interesting condition (see Section 5). In this context, it is plausible to consider the difference between two distinct processes, one with greater overdispersion for the positive part (which we will model as geometric) and another with lower variability for the negative part (which we will model as Poisson). The new process can be used very well for modeling the change in price of a commodity, stock or other financial product.

The rest of the paper unfolds as follows. In Section 2, the NSINAR(1) process is introduced. In Section 3, some of its properties are outlined. Estimation methods for the model parameters are proposed, while numerical results from Monte Carlo simulation experiments are presented and discussed in Section 4. An application to a real data set is addressed in Section 5. Finally, in Section 6, we offer some concluding remarks.

2. NSINAR(1) process

Let $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{R}$ denote the set of non-negative integers, integers and real numbers, respectively. All random variables will be defined on the same probability space. In this section, we introduce a stationary first-order integer-valued autoregressive process with geometric–Poisson marginals. With this aim, we first review the new geometric first-order integer-valued autoregressive (NGINAR(1)) process by Ristić et al. [17] and the Poisson INAR(1) process by Al-Osh and Alzaid [1].

The NGINAR(1) process is defined such that

$$X_t = \alpha * X_{t-1} + \epsilon_t, \quad t \in \mathbb{Z},$$

where "$*$" denotes the negative binomial thinning operator [17]. This operator is defined by $\alpha * X \stackrel{d}{=} \sum_{i=1}^{X} W_i$, where the symbol "$\stackrel{d}{=}$" means "has the same distribution as" and $\{W_i\}_{i=1}^{\infty}$ is a sequence of independent and identically distributed (i.i.d.) random variables, independent of X, following a geometric distribution with mean $\alpha \in [0, 1)$. Also, $\{\epsilon_t\}_{t\in\mathbb{Z}}$ is a sequence of i.i.d. random variables independent of $\{W_i\}_{i=1}^{\infty}$, with ϵ_t and X_{t-l} being independent for all $l \geq 1$ and distributed such that $\{X_t\}_{t\in\mathbb{Z}}$ is a stationary process having geometric marginals with probability function assuming the form $\Pr(X_t = x) = \mu^x/(1+\mu)^{x+1}$, for all non-negative integers x, where $\mu > 0$.

Ristić et al. [17] proved that the probability mass function of ϵ_t is given by

$$\Pr(\epsilon_t = l) = \left(1 - \frac{\alpha\mu}{\mu-\alpha}\right)\frac{\mu^l}{(1+\mu)^{l+1}} + \frac{\alpha\mu}{\mu-\alpha}\,\frac{\alpha^l}{(1+\alpha)^{l+1}}, \quad l \in \mathbb{N}, \tag{1}$$

with the condition $\alpha \leq \mu/(1+\mu)$ being necessary to guarantee that all probabilities in (1) are non-negative. The distribution of the random variable ϵ_t is therefore a mixture of two independent geometric distributions with means μ and α. For more details about the NGINAR(1) process, see [17,3,14,15].

The Poisson INAR(1) process introduced by Al-Osh and Alzaid [1] is defined on the discrete support $\mathbb{N}$ of nonnegative integers by means of the difference equation

$$Y_t = \alpha \circ Y_{t-1} + \varepsilon_t, \quad t \in \mathbb{Z}, \tag{2}$$