Statistics and Probability Letters 109 (2016) 184–188

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

Concomitants of records: Limit results, generation techniques, correlation

Alexei Stepanov[a], Alexandre Berred[b,*], Valery B. Nevzorov[c]

[a] Immanuel Kant Baltic Federal University, A.Nevskogo 14, Kaliningrad, 236041, Russia

[b] Faculté des Sciences et Techniques, Université du Havre, B.P. 540, 76058 Le Havre Cedex, France

[c] Department of Probability and Statistics, St.Petersburg State University, University Prospect 28, St.Petersburg, 198504, Russia

ARTICLE INFO

Article history:
Received 24 June 2015
Received in revised form 19 November 2015
Accepted 20 November 2015
Available online 28 November 2015

Keywords:
Bivariate distributions
Records
Concomitants
Strong limit laws
Rank correlation coefficients
Generation techniques

ABSTRACT

In this paper, we investigate concomitants of record values. We first study their distributional properties and derive corresponding strong limit theorems. We then propose algorithms to generate them. We finally consider a new rank correlation coefficient based on concomitants of records.

1. Introduction

Let $(X, Y), (X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n), \ldots$ be independent and identically distributed random vectors with continuous bivariate distribution $F(x, y)$ and corresponding marginal distributions $H(x)$ and $G(y)$. In the case of the existence of the corresponding densities, they will be denoted by $f(x, y)$, $h(x)$ and $g(y)$, respectively. For the sequence of X's, the sequences of record values $X(n)$ and record times $L(n)$ are defined by

$$L(0) = 0, \qquad L(1) = 1,$$
$$L(n+1) = \min\{j > L(n) : X_j > X_{L(n)}\},$$
$$X(n) = X_{L(n)}, \quad n = 1, 2, \ldots.$$

For the classical properties of record values and times refer, among others, to Nevzorov (2001). Let $m \geq 1$ be such that $X(k) = X_m$. Then the random variable $Y[k] = Y_m$ is called the concomitant of the record value $X(k)$. The concept of concomitants of records was first proposed in Houchens (1984). Concomitants of records can be of great interest in various applications such as insurance, survival analysis, and biostatistics.

Distributional properties of concomitants of records were investigated in Nevzorov and Ahsanullah (2000). In particular, it was shown that

$$f_{Z[1],\ldots,Z[n]}(x_1, y_1, \ldots, x_n, y_n) = \frac{f(x_1, y_1)}{1 - H(x_1)} \cdots \frac{f(x_{n-1}, y_{n-1})}{1 - H(x_{n-1})} f(x_n, y_n) \quad (l_H < x_1 < \cdots < x_n < r_H, y_i \in \mathbb{R}), \tag{1.1}$$

* Corresponding author.
E-mail addresses: alexeistep45@mail.ru (A. Stepanov), alexandre.berred@univ-lehavre.fr (A. Berred), valnev@mail.ru (V.B. Nevzorov).

http://dx.doi.org/10.1016/j.spl.2015.11.020
0167-7152/

where $Z[1]=(X(1),Y[1]),\ldots,Z[n]=(X(n),Y[n])$ and $l_H=\inf\{x\in R: H(x)>0\}$ and $r_H=\sup\{x\in R: H(x)<1\}$ are the left and right bounds of the support of H, respectively. Similarly, the left and right bounds of the support of G are also denoted by l_G and r_G. By integrating (1.1), we obtain the joint density of $Y[1],\ldots,Y[n]$:

$$f_{Y[1],\ldots,Y[n]}(y_1,\ldots,y_n)=\int\cdots\int_{l_H<x_1<\cdots<x_n<r_H}\frac{f(x_1,y_1)}{1-H(x_1)}\cdots\frac{f(x_{n-1},y_{n-1})}{1-H(x_{n-1})}f(x_n,y_n)dx_1\ldots dx_n. \tag{1.2}$$

Bairamov and Stepanov (2010, 2011) considered the following limit:

$$\lim_{x\to r_H^-}\frac{G(y)-F(x,y)}{1-H(x)}=\beta(y)\in[0,1]. \tag{1.3}$$

They used (1.3) to analyze the asymptotic behavior of the concomitants of records and top order statistics. A classification of bivariate distributions by values of β is as follows.

(i) Assume that for some $c\in[-\infty,\infty]$,

$$\beta(y)=0\quad(y<c)\quad\text{and}\quad\beta(y)=1\quad(y>c).$$

Then the distribution F is said to be a c-stable record-concomitant distribution.

(ii) If such c does not exist, then F is said to be an unstable record-concomitant distribution. The following lemma was given in Bairamov and Stepanov (2011).

Lemma 1.1. *Assume that the limit in (1.3) exists. Then*

$$P(Y[n]\le y)\to\beta(y)\quad(n\to\infty).$$

It follows from Lemma 1.1 that if F is an unstable record-concomitant distribution, then there is a non-degenerate limiting distribution for $Y[n]$. Naturally, $Y[n]$ does not converge in probability to a constant. On the other hand, when F is a c-stable record-concomitant distribution, then $Y[n]\xrightarrow{p}c$.

Now we present recent generalizations of the Borel–Cantelli lemma obtained in Stepanov (2014). These generalizations will be used in Section 2 to derive strong limit results for concomitants of records. Suppose that $A_1,A_2,\ldots$ is a sequence of events defined on a common probability space and that A_i^c denotes the complement of the event A_i. Let i.o. be an abbreviation for "infinitively often". We also define the Markov sequences of events.

Definition 1.1. We say that A_n $(n\ge1)$ is a Markov sequence of events if the sequence of random variables I_{A_n} $(n\ge1)$ is a Markov chain.

Lemma 1.2. *Let $A_1,A_2,\ldots$ be a Markov sequence of events such that $P(A_n)\to0$. Consider the series*

$$\sum_{n=1}^{\infty}P(A_n^cA_{n+1}). \tag{1.4}$$

If the series in (1.4) is convergent, then $P(A_n\ i.o.)=0$. If the series in (1.4) is divergent, then $P(A_n\ i.o.)=1$.

Lemma 1.3. *Let $A_1,A_2,\ldots$ be a Markov sequence of events such that $P(A_n)\to0$. Consider the series*

$$\sum_{n=1}^{\infty}P(A_nA_{n+1}^c). \tag{1.5}$$

If the series in (1.5) is convergent, then $P(A_n\ i.o.)=0$. If the series in (1.5) is divergent, then $P(A_n\ i.o.)=1$.

The rest of the paper is as follows. In Section 2, we derive strong limit results for concomitants of records for the case when F is a c-stable record-concomitant distribution, i.e. when $Y[n]\xrightarrow{p}c$. In Section 3, we propose algorithms for generating concomitants of record values. Finally, in Section 4, we discuss a new rank correlation coefficient based on concomitants of records.

2. Strong limit theorems for concomitants

In this section, we present strong limit results for concomitants of records.