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Confidence ellipsoids for the primary regression coefficients in two seemingly unrelated regression models

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ABSTRACT

We derive two new confidence ellipsoids (*CE*s) and four *CE* variations for covariate coefficient vectors with nuisance parameters under the seemingly unrelated regression (*SUR*) model. Unlike most *CE* approaches for *SUR* models studied so far, we assume unequal regression coefficients for our two regression models. The two new basic *CE*s are a *CE* based on a Wald statistic with nuisance parameters and a *CE* based on the asymptotic normality of the *SUR* two-stage unbiased estimator of the primary regression coefficients. We compare the coverage and volume characteristics of the six *SUR*-based *CE*s via a Monte Carlo simulation. For the configurations in our simulation, we determine that, except for small sample sizes, a *CE* based on a two-stage statistic with a Bartlett corrected $(1-\alpha)$ percentile is generally preferred because it has essentially nominal coverage and relatively small volume. For small sample sizes, the parametric bootstrap *CE* based on the two-stage estimator attains close-to-nominal coverage and is superior to the competing *CE*s in terms of volume. Finally, we apply three *SUR* Wald-type *CE*s with favorable coverage properties and relatively small volumes to a real data set to demonstrate the gain in precision over the ordinary-least-squares-based *CE*.

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1. Introduction

Zellner [26] first introduced *seemingly unrelated regression* (*SUR*) equations as a way to reduce the variability of coefficient estimators in a linear model. He and others, see [9], have shown that the reduction in variance increases as the correlation between the error terms of the equations increases. Many types of *SUR* models have been proposed, the most common of which is the linear *SUR* model given by Zellner [26]; however, Liu [9] has considered a multivariate *SUR* model where the response is a matrix instead of a vector. Timm [21] has considered a *MANOVA* model, which generalizes the *SUR* models, and Revankar [15,16] has examined properties of estimators that use residuals as estimates of the variance–covariance matrix. Moreover, Srivastava and Srivastava [20] have used a convex combination of the ordinary-least-squares (*OLS*) estimator and a *SUR* estimator to arrive at an estimator that conditionally has smaller variance than the *OLS* and *SUR* estimators.

Drton and Richardson [3] and Drton [2] have shown that multimodality may exist in the *SUR* likelihood function, and Foschi and Kongoghiorghes [4] have derived estimators of parameters in the *SUR* model when sample sizes of the dependent variables are unequal. Liu [9] has considered estimation of the *SUR* model when the error terms have a general elliptical distribution, and Verzilli et al. [23] have used a Bayesian *SUR* approach to model multivariate quantitative traits in genetic association studies. Also, King [8] and Winkelmann [25] have derived count-data *SUR* models, and Gallant [6] has introduced the general seemingly unrelated nonlinear regression model. In addition, Srivastava and Giles [19] have provided a reference to the many topics of *SUR* models.

Although Phillips [13] has derived the exact distribution of the *SUR* estimator, the distribution is complicated and does not lend itself to easily-formed inferential methods. Thus, asymptotic methods are of practical use. In particular, *maximum likelihood estimation* (*MLE*) methods are frequently applied. The appropriate *SUR*-based *MLE*s are easily determined if one uses an iterative procedure proposed by Park [12]. For small sample sizes, *MLE*-based confidence intervals and ellipsoids have poor coverage properties. Rocke [18] has used bootstrapping in hypothesis testing while Rilstone and Veall [17] have used bootstrapping to construct confidence intervals when the regression coefficients are equal. These authors have proposed these bootstrap methods to remedy the problems associated with small sample sizes. Fraser et al. [5] have applied a highly accurate likelihood third-order interval estimation method that works for small samples, but this procedure is applicable only for scalar parameters and not for vector parameters. Thus, we need simple, accurate methods to estimate a parameter vector with a confidence region. For the two-equation *SUR* model, Kariya [7] has tested for the independence of two *SUR* equations, and Wang and Sun [24] have employed Bayesian methods for the correlation coefficient in two *SUR* equations. Furthermore, Ma and Ye [11] have presented an efficient improved estimation for the case of two *SUR* models.

Here, we derive and compare two new *CE*s and four *CE*s for the principal model coefficients in the *SUR* model when the regression coefficients are unequal. We remark that we could find no literature on *SUR CE*s that are direct competitors. First, we derive a new matrix-derivative-based formulation of Fisher's information matrix for the covariate parameters that, unlike the point-wise representation given in [19], can be conveniently utilized so that we can formulate a Wald statistic for *CE* estimation. Next, we develop the *CE*s for the primary regression coefficients of interest assuming a two-equation *SUR* model. The first basic *SUR CE* is formed from a Wald statistic and the usual $(1-\alpha)$ percentile for the appropriate chi-squared distribution. A related *SUR CE* is based on the Wald statistic and a Bartlett-corrected $(1-\alpha)$ percentile for the $(1-\alpha)$ chi-squared distribution. Another related *SUR CE* is formed from the Wald statistic and a parametric bootstrap estimate of its $(1-\alpha)$ percentile. The second basic *SUR CE* is formed from an asymptotically normally distributed two-stage estimator, and a related *SUR CE* is formed from this asymptotically normally distributed two-stage estimator and a Bartlett-corrected $(1-\alpha)$ percentile estimate. The final *SUR CE* that we consider is based on a parametric bootstrap of a two-stage *SUR* coefficient estimator.

Using a Monte Carlo simulation study, we demonstrate that employing a bootstrap adjusted percentile estimate with a two-stage-estimator-based statistic, one can produce *CE*s that perform relatively well in the small sample-size case, and, in particular, perform well in the small sample-size case when correlation is present under the *SUR* model. Specifically, we show that for small sample sizes, the parametric bootstrap two-stage-based statistic displayed nominal coverage while other