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An L_p-theory for stochastic partial differential equations driven by Lévy processes with pseudo-differential operators of arbitrary order

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Abstract

In this article we present uniqueness, existence, and L_p-estimates of the quasilinear stochastic partial differential equations driven by Lévy processes of the type

$$du = (Lu + F(u))dt + G^k(u)dZ_t^k, \tag{0.1}$$

where L is a pseudo-differential operator and Z^k are independent Lévy processes ($k = 1, 2, \cdots$). The operator L is random and may depend also on time and space variables. In particular, our results include an L_p-theory of $2m$-order SPDEs with coefficients measurable in (ω, t) and continuous in x.

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Keywords: Stochastic partial differential equations driven by Lévy processes; L_p-theory; Pseudo-differential operator; High-order operators

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1. Introduction

The maximal L_p-regularity theory for stochastic partial differential equations of the type (0.1) has been developed by many mathematicians. Krylov [14] initiated the theory for the second-order linear SPDEs. After his work, the theory has been extended to more general operators. For instance, see [9,2] for lower order non-local operators and [19] for $2m$-order operators.

In this article we further extend these results to SPDEs driven by Lévy processes with pseudo-differential operator of arbitrary order without any smoothness condition of the operator with respect to (ω, t). We assume that the pseudo-differential operator L depends on (ω, t) and its symbol $\psi(\omega, t, \xi)$ satisfies the following conditions: there exist constants $\gamma, \kappa > 0$ so that

$$\Re[-\psi(\omega, t, \xi)] \geq \kappa |\xi|^{\gamma}, \tag{1.1}$$

and for any multi-index α, $|\alpha| \leq \left\lfloor \frac{d}{2} \right\rfloor + 1$,

$$|D_{\xi}^{\alpha} \psi(\omega, t, \xi)| \leq \kappa^{-1} |\xi|^{\gamma - |\alpha|}. \tag{1.2}$$

Conditions (1.1) and (1.2) are fulfilled by a large class of pseudo-differential operators. For instance, if L_1 and L_2 satisfy (1.1) and (1.2) with γ_1 and γ_2 respectively and if the symbols of L_1 and L_2 are real valued then for any constants $a, b > 0$ the operator $(-L_1)^a(-L_2)^b$ satisfies the conditions with $\gamma = a\gamma_1 + b\gamma_2$. We emphasize that the operator L is merely measurable in (ω, t), the driving noises are Lévy processes, and the order of L can be arbitrary positive real number.

As an application of our result for pseudo-differential operators, we obtain an L_p-theory for $2m$-order SPDE with the operator $L = \sum_{|\alpha|=|\beta|=m} D_x^{\alpha} D_x^{\beta} a^{\alpha\beta}(\omega, t, x)$, which is measurable in (ω, t) and continuous in x. To the best of our knowledge, the first article which introduced an L_p-regularity result for $2m$-order SPDEs is [19], where the leading coefficients are assumed to be continuous in (t, x) and the driving noises are Wiener processes. In our article we do not assume such continuity of $a^{\alpha\beta}$ in t, and furthermore our equation is driven by Lévy processes. More specifically, we prove uniqueness and existence results of the equation

$$du = \left((-1)^{m-1} \sum_{|\alpha|=|\beta|=m} a^{\alpha\beta} D^{\alpha} D^{\beta} u + F(u) \right) dt + G_c^k(u) dW_t^k + G_d^k(u) Z_t^k$$

in the space $\mathbb{H}_p^{2m}(T) := L_p(\Omega \times (0, T); H_p^{2m})$ under some appropriate conditions on the non-linear terms F, G_c and G_d (see Assumptions 2.9, 2.10, and 2.11). Here W_t^k, $k = 1, 2, \ldots$, are independent Wiener processes independent of Lévy processes Z_t^k. It is also proved that for this solution it holds that

$$\begin{aligned} \|u\|_{\mathbb{H}_p^{2m}(T)}^p \leq C \Big(&\|F(0)\|_{\mathbb{L}_p(T)}^p + \|G_c(0)\|_{\mathbb{H}_p^m(T, l_2)}^p + \|G_d(0)\|_{\mathbb{H}_p^{2(1-1/p)m}(T, l_2)}^p \\ &+ \mathbf{E}\|u(0)\|_{H_p^{2m(1-1/p)}}^p \Big). \end{aligned}$$

Furthermore, we obtain a general regularity theory, i.e., $u \in \mathbb{H}_p^{2m+n}(T)$, here $n \in \mathbb{R}$, if F, G_c, and G_d satisfy a certain regularity condition related to parameter n. See Section 2 for notation and details.

So far, L_p-theories for SPDEs have been focused mostly on SPDEs with continuous processes, and only few results are related to Lévy processes. An L_p-theory for SPDEs driven by Lévy processes can be found [3,9,8,16,17]. The operators considered in [3,8,9,16,17] are the