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The sequential empirical process of a random walk in random scenery

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Abstract

A random walk in random scenery $(Y_n)_{n\in\mathbb{N}}$ is given by $Y_n = \xi_{S_n}$ for a random walk $(S_n)_{n\in\mathbb{N}}$ and i.i.d. random variables $(\xi_n)_{n\in\mathbb{Z}}$. In this paper, we will show the weak convergence of the sequential empirical process, i.e. the centered and rescaled empirical distribution function. The limit process shows a new type of behavior, combining properties of the limit in the independent case (roughness of the paths) and in the long range dependent case (self-similarity).

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1. Introduction

For a stationary, real valued sequence $(Y_n)_{n\in\mathbb{N}}$ of random variables with marginal distribution function F, the empirical distribution function F_n is defined by

$$F_n(t) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{\{Y_i \le t\}}. \tag{1}$$

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If the marginal distribution function F is continuous, we can without loss of generality assume that $F(t) = t$ (otherwise replacing Y_n by $F(Y_n)$). The sequential empirical process is a two-parameter stochastic process $\big(W_n(s,t)\big)_{s,t\in[0,1]}$ defined by

$$W_n(s,t) = \sum_{i=1}^{[ns]} \left(\mathbb{1}_{\{Y_i \le t\}} - t\right), \tag{2}$$

where $[x]$ denotes the integer part of x. Note that we will have to rescale this process in order to obtain weak convergence, but as we need a different scaling for different kinds of stochastic processes, we have not included the scaling here. For i.i.d. (independent and identical distributed) random variables $(Y_n)_{n\in\mathbb{N}}$, Donsker [7] showed the weak convergence of the (non-sequential) empirical process $\big(\frac{1}{\sqrt{n}} W_n(1,t)\big)_{t\in[0,1]}$ to a Brownian bridge. This was extended by Müller [18] to the sequential empirical process $\big(\frac{1}{\sqrt{n}} W_n(s,t)\big)_{s,t\in[0,1]}$. The limit Gaussian process is the so called Kiefer–Müller process K, which is self-similar with exponent $b = \frac{1}{2}$, that means for any $a > 0$ the process $\big(K(as,t)\big)_{s,t\in[0,1]}$ has the same distribution as $\big(a^{\frac{1}{2}} K(s,t)\big)_{s,t\in[0,1]}$. For fixed $s \in [0,1]$, $(K(s,t))_{t\in[0,1]}$ is a Brownian bridge, while for fixed $t \in [0,1]$ $(K(s,t))_{s\in[0,1]}$ is a Brownian motion. This implies that there is an almost surely continuous modification of K, but the paths are not γ-Hölder continuous for any $\gamma > \frac{1}{2}$.

This limit theorem has been extended to different kinds of short range dependent processes $(Y_n)_{n\in\mathbb{N}}$, where one still needs a $n^{-\frac{1}{2}}$ scaling and the limit process is still self-similar with exponent $\frac{1}{2}$. For example, Berkes and Philipp [2] studied approximating functionals of strongly mixing sequences and Berkes, Hörmann, Schauer [1] so called S-mixing random variables. In the short range dependent case, the limit process is for fixed $t \in [0,1]$ a Brownian motion as in the independent case, so the paths are not smoother.

For long range dependent processes, the limit behavior is different in many aspects. For Gaussian sequences with slowly decaying covariances, Dehling and Taqqu [6] showed the convergence of sequential empirical process to a limit process that is self-similar with exponent $b > \frac{1}{2}$ and that is degenerate in the following sense: For fixed s, the process is not a Brownian bridge, but a deterministic function multiplied by a random variable. The paths for fixed s might be differentiable. For fixed t, the limit process is a fractional Brownian motion which is γ-Hölder continuous with exponent $\gamma > \frac{1}{2}$. For long range dependent linear processes, analog results were proved by Ho and Hsing [13].

In this paper, we will consider the random walk in random scenery, which is often considered to be another model for a long range dependent sequence of random variables. Let $(S_n)_{n\in\mathbb{N}}$ with $S_n = \sum_{i=1}^{n} X_i$ be a random walk in the normal domain of attraction of an α-stable Lévy process (with i.i.d., integer valued increments $(X_n)_{n\in\mathbb{N}}$) and $(\xi_n)_{n\in\mathbb{Z}}$ a sequence of i.i.d. random variables (called scenery). Then the stationary process $(Y_n)_{n\in\mathbb{N}}$ with $Y_n = \xi_{S_n}$ is called random walk in random scenery and was first investigated by Kesten and Spitzer [14] and Borodin [4].

The behavior of partial sum process Z_n with $Z_n(s) = \sum_{i=1}^{[ns]} Y_i$ has been studied extensively. It converges weakly to a self-similar process with exponent $b > \frac{1}{2}$, which has smooth paths even if the random variables $(\xi_n)_{n\in\mathbb{Z}}$ are in the domain of attraction of a Lévy process with jumps, see [14]. Other results include the law of the iterated logarithm (Khoshnevisan and Lewis [15]), large deviations (Gantert, König, and Shi [10]), extremes (Franke and Saigo [9]) and U-statistics (Guillotin-Plantard and Ladret [12], Franke, Pène, and Wendler [8]). As far as we know, there are no results on the empirical process of a random walk in random scenery.