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Statistical inference for time-changed Lévy processes via Mellin transform approach

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Abstract

Given a Lévy process $(L_t)_{t\geq 0}$ and an independent nondecreasing process (time change) $(\mathscr{T}(t))_{t\geq 0}$, we consider the problem of statistical inference on $\mathscr{T}$ based on low-frequency observations of the time-changed Lévy process $L_{\mathscr{T}(t)}$. Our approach is based on the genuine use of Mellin and Laplace transforms. We propose a consistent estimator for the density of the increments of $\mathscr{T}$ in a stationary regime, derive its convergence rates and prove the optimality of the rates. It turns out that the convergence rates heavily depend on the decay of the Mellin transform of $\mathscr{T}$. Finally, the performance of the estimator is analysed via a Monte Carlo simulation study.

Keywords: Time-changed Lévy processes; Low-frequency observations; Mellin transform; Laplace transform

1. Introduction

Let $L = (L_t)_{t\geq 0}$ be a one-dimensional Lévy process with a Lévy triplet (μ, σ^2, ν) and let $\mathscr{T} = (\mathscr{T}(s))_{s\geq 0}$ be a non-negative, non-decreasing stochastic process independent of L with $\mathscr{T}(0) = 0$. A time-changed Lévy process $Y = (Y_s)_{s\geq 0}$ is then defined via $Y_s = L_{\mathscr{T}(s)}$. The process $\mathscr{T}$ is usually referred to as time change. Here we consider the problem of statistical

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inference on the distribution of the time change $\mathscr{T}$ based on low-frequency observations of the time-changed Lévy process (Y_t). Suppose that n observations of the time-changed Lévy process (Y_t) at times $t_j = j\Delta,\ j = 0, \ldots, n$, are available. If the sequence $\mathscr{T}(t_j) - \mathscr{T}(t_{j-1}),\ j = 1, \ldots, n$, is ergodic, strictly stationary with invariant stationary distribution p_Δ, then for any bounded "test function" f,

$$\frac{1}{n}\sum_{j=1}^{n} f\left(L_{\mathscr{T}(t_j)} - L_{\mathscr{T}(t_{j-1})}\right) \xrightarrow{\text{a.s.}} \mathbb{E}_{p_\Delta}[f(L_{\mathscr{T}(\Delta)})], \quad n \to \infty. \tag{1}$$

The limiting expectation in (1) is then given by

$$\mathbb{E}_{p_\Delta}[f(L_{\mathscr{T}(\Delta)})] = \int_0^\infty \mathbb{E}[f(L_s)]\, p_\Delta(ds).$$

Taking $f(z) = f_u(z) = \exp(\mathrm{i}uz),\ u \in \mathbb{R}$, and using the well-known Lévy–Khintchine formula:

$$\phi(u) := \mathbb{E}\left[e^{\mathrm{i}uL_t}\right] = e^{-t\psi(u)}, \quad u \in \mathbb{R},\ t \geq 0$$

with

$$\psi(u) := \frac{1}{2}\sigma^2 u^2 - \mathrm{i}\mu u - \int_{\mathbb{R}} \left(e^{ixu} - 1 - \mathrm{i}xu1_{\{|x|\leq 1\}}(x)\right) \nu(dx),$$

we arrive at the following representation for the characteristic function of Y_Δ:

$$\mathbb{E}\left[\exp\left(\mathrm{i}uY_\Delta\right)\right] = \int_0^\infty \exp(-t\psi(u))\, p_\Delta(dt) = \mathscr{L}[p_\Delta](\psi(u)), \tag{2}$$

where $\mathscr{L}[p_\Delta]$ stands for the Laplace transform of p_Δ. Hence the problem of statistical inference on p_Δ is related to the problem of Laplace transform inversion based on noisy and indirect (due to the presence of ψ) observations. The resulting statistical inverse problem is known to be highly nonlinear and ill-posed, see [11]. Here we propose a novel and general approach for the estimation of p_Δ, which is based on the genuine use of Laplace and Mellin transforms.

The problem of estimating the parameters of a discretely observed Lévy process has recently got much attention in the literature (see, e.g., a recent monograph [4]). Time-changed Lévy processes have been recently studied in Belomestny [3], where it is shown how to estimate the Lévy triplet of the underlying Lévy process L from low-frequency observations of the process (Y_t) without knowledge of the time change $\mathscr{T}$. The results in [3] rely on the fact that the process L is essentially multidimensional. To the best of our knowledge, the problem of estimating the time change $\mathscr{T}$ has not yet been studied in the literature except in some special cases. For example, the case of stopped Poisson process was considered in a recent paper of Comte and Genon-Catalot [9]. The case of the time changed Brownian motion (the so-called statistical Skorohod embedding problem) has recently been studied in Belomestny and Schoenmakers [5]. Note that the latter problem can be transformed to a kind of deconvolution problem using time scalability of Brownian motion. Unfortunately, such a transformation is not possible in the case of general Lévy processes. Statistical inference for time-changed Lévy processes based on high-frequency observations of (Y_t) was the subject of many studies, see, e.g. Bull, [8] and Todorov and Tauchen, [17] and the references therein. Although the problem of estimating the density of $\mathscr{T}$ from discrete (low-frequency) observations of the corresponding time-changed Lévy process Y is related to the problem of non-parametric mixture estimation (see, e.g. Zhang [19] for continuous case or Roueff and Rydén [16] for discrete mixtures), it does not, in general, fit