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On degenerate linear stochastic evolution equations driven by jump processes

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Abstract

We prove the existence and uniqueness of solutions of degenerate linear stochastic evolution equations driven by jump processes in a Hilbert scale using the variational framework of stochastic evolution equations and the method of vanishing viscosity. As an application of this result, we derive the existence and uniqueness of solutions of degenerate parabolic linear stochastic integro-differential equations (SIDEs) in the Sobolev scale. The SIDEs that we consider arise in the theory of non-linear filtering as the equations governing the conditional density of a degenerate jump–diffusion signal given a jump–diffusion observation, possibly with correlated noise.

Keywords: Systems of stochastic integro-differential equations; L^2 theory; Degenerate stochastic parabolic PDEs; Levy processes

1. Introduction

Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a probability space with the filtration $\mathbf{F} = (\mathcal{F}_t)_{0 \le t \le T}$ of sigma-algebras satisfying usual conditions. In a triple of Hilbert spaces $(H^{\alpha+\mu}, H^{\alpha}, H^{\alpha-\mu})$ with parameters $\mu \in (0, 1]$ and $\alpha \geq \mu$, we consider a linear stochastic evolution equation given by

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$$du_t = (\mathcal{L}_t u_t + f_t)\,dV_t + (\mathcal{M}_t u_{t-} + g_t)\,dM_t, \quad t \le T, \tag{1}$$
$$u_0 = \varphi,$$

where V_t is a continuous non-decreasing process, M_t is a cylindrical square integrable martingale, $\mathcal{L}$ and $\mathcal{M}$ are linear adapted operators, and ϕ, f, and g are adapted input functions.

By virtue of Theorems 2.9 and 2.10 in [7], under some suitable conditions on the data φ, f and g, if $\mathcal{L}$ satisfies a growth assumption and $\mathcal{L}$ and $\mathcal{M}$ satisfy a coercivity condition in the triple $(H^{\alpha+\mu}, H^\alpha, H^{\alpha-\mu})$, then there exists a unique solution $(u_t)_{t\le T}$ of (1) that is strongly càdlàg in H^α and belongs to $L^2(\Omega\times[0,T], \mathcal{O}_T, dV_t d\mathbf{P}; H^{\alpha+\mu})$, where $\mathcal{O}_T$ is the optional sigma-algebra on $\Omega\times[0,T]$. In this paper, under a weaker assumption than coercivity (see Assumption 2.2 (α,μ)) and using the method of vanishing viscosity, we prove that there exists a unique solution $(u_t)_{t\le T}$ of (1) that is strongly càdlàg in $H^{\alpha'}$ for all $\alpha' < \alpha$ and belongs to $L^2(\Omega\times[0,T], dV_t d\mathbf{P}; H^\alpha)$. Furthermore, under some additional assumptions on the operators $\mathcal{L}$ and $\mathcal{M}$ we can show that the solution u is weakly càdlàg in H^α.

The variational theory of deterministic degenerate linear elliptic and parabolic PDEs was established by O.A. Oleĭnik and E.V. Radkevich in [18,19]. É. Pardoux, in [20], developed the variational theory of monotone stochastic evolution equations, which was extended in [11,12,8,7] by N.V. Krylov, B.L. Rozovskiĭ, and I. Gyöngy. Degenerate parabolic stochastic partial differential equations (SPDEs) driven by continuous noise were first investigated by N.V. Krylov and B.L. Rozovskiĭ in [13]. These types of equations arise in the theory of non-linear filtering of continuous diffusion processes as the Zakai equation and as equations governing the inverse flow of continuous diffusions. In [5], the solvability of systems of linear SPDEs in Sobolev spaces was proved by M. Gerencsér, I. Gyöngy, and N.V. Krylov, and a small gap in the proof of the main result of [13] was fixed. In Chapters 2, 3, and 4 of [23], B.L. Rozovskiĭ offers a unified presentation and extension of earlier results on the variational framework of linear stochastic evolution systems and SPDEs driven by continuous martingales (e.g. [20,11–13]). Our existence and uniqueness result on degenerate linear stochastic evolution equations driven by jump processes (Theorem 3.3) extends Theorem 2 in Chapter-3—Section 2.2 of [23] to include the important case of equations driven by jump processes. It is also worth mentioning that the semigroup approach for non-degenerate SPDEs driven by Lévy processes is well-studied (see, e.g. [21,22]).

As a special case of (1), we will consider a system of stochastic integro-differential equations. Before introducing the equation, let us describe our driving processes. Let $\mathcal{P}_T$ and $\mathcal{R}_T$ be the predictable and progressive sigma-algebras on $\Omega\times[0,T]$, respectively. Let $\eta(dt,dz)$ be an integer-valued random measure on $(\mathbf{R}_+\times Z, \mathcal{B}(\mathbf{R}_+)\otimes\mathcal{Z})$ with predictable compensator $\pi_t(dz)dV_t$. Let $\tilde\eta(dt,dz)=\eta(dt,dz)-\pi_t(dz)dt$ be the martingale measure corresponding to $\eta(dt,dz)$. Let $(Z^2,\mathcal{Z}^2)$ be a measurable space with $\mathcal{R}_T$-measurable family $\pi_t^2(dz)$ of sigma-finite random measures on Z. Let $w_t = (w_t^\varrho)_{\rho\in\mathbf{N}}, t\ge 0$, be a sequence of continuous local uncorrelated martingales such that $d\langle w^\varrho\rangle_t = dV_t$, for all $\varrho\in\mathbf{N}$. Let $d_1, d_2\in\mathbf{N}$. For convenience, we set $(Z^1,\mathcal{Z}^1)=(Z,\mathcal{Z})$ and $\pi_t^1=\pi_t$. We consider the d_2-dimensional system of SIDEs on $[0,T]\times\mathbf{R}^{d_1}$ given by

$$\begin{aligned} du_t^l &= \left((\mathcal{L}_t^{1;l}+\mathcal{L}_t^{2;l})u_t + b_t^i\partial_i u_t^l + c_t^{l\bar l}u_t^{\bar l}(x) + f_t^l\right)dV_t + (\mathcal{N}_t^{l\varrho}u_t + g_t^{l\varrho})dw_t^\varrho \\ &\quad + \int_{Z^1}\left(\mathcal{I}_{t,z}^l u_{t-}^{\bar l} + h_t^l(z)\right)\tilde\eta(dt,dz), \\ u_0^l &= \varphi^l, \quad l\in\{1,\dots,d_2\}, \end{aligned} \tag{2}$$