Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications 125 (2015) 1653–1687

stochastic processes and their applications

www.elsevier.com/locate/spa

Speed of convergence for laws of rare events and escape rates

Ana Cristina Moreira Freitas[a], Jorge Milhazes Freitas[b,*], Mike Todd[c]

[a] *Centro de Matemática & Faculdade de Economia da Universidade do Porto, Rua Dr. Roberto Frias, 4200-464 Porto, Portugal*

[b] *Centro de Matemática & Faculdade de Ciências da Universidade do Porto, Rua do Campo Alegre 687, 4169-007 Porto, Portugal*

[c] *Mathematical Institute, University of St Andrews, North Haugh, St Andrews, KY16 9SS, Scotland, United Kingdom*

Received 6 August 2014; received in revised form 30 October 2014; accepted 14 November 2014
Available online 24 November 2014

Abstract

We obtain error terms on the rate of convergence to Extreme Value Laws, and to the asymptotic Hitting Time Statistics, for a general class of weakly dependent stochastic processes. The dependence of the error terms on the 'time' and 'length' scales is very explicit. Specialising to data derived from a class of dynamical systems we find even more detailed error terms, one application of which is to consider escape rates through small holes in these systems.

MSC: 37A50; 60G70; 37B20; 60G10; 37C25

Keywords: Extreme value theory; Return time statistics; Stationary stochastic processes; Metastability

* Corresponding author.
E-mail addresses: amoreira@fep.up.pt (A.C.M. Freitas), jmfreita@fc.up.pt (J.M. Freitas), mjt20@st-andrews.ac.uk (M. Todd).
URLs: http://www.fep.up.pt/docentes/amoreira/ (A.C.M. Freitas), http://www.fc.up.pt/pessoas/jmfreita/ (J.M. Freitas), http://www.mcs.st-and.ac.uk/~miket/ (M. Todd).

http://dx.doi.org/10.1016/j.spa.2014.11.011
0304-4149/

1. Introduction

The study of the statistics of extreme events is both of classical importance, and a crucial topic across contemporary science. Classically, the underlying stochastic processes are assumed to be independently distributed, but many more recent developments in this topic relate to the study of Extreme Value Laws (EVL) for dependent systems. A standard approach to prove the existence of EVL in this setting is to check some conditions on the underlying process, for example Leadbetter's conditions $D(u_n)$ and $D'(u_n)$ in [33]. Inspired by [13], in a series of works [20,21,23,24] the authors developed these conditions so that they had wider application, the main motivation being to stochastic processes coming from dynamical systems. A natural question to now ask is: how fast is the convergence to the EVL? (To rephrase, at a given finite stage in the process, what is the difference, or error term, between the law observed up to this time and the asymptotic law?) For example, if the convergence were to be very slow, in a simulation the laws would be essentially invisible. In this paper we address this question, particularly in the context of the latter papers above.

Error terms in the i.i.d. context are rather well-known, see for example [26,43] and the discussion in [37, Section 2.4]. However, the literature on dependent processes is much less extensive (for one case, see [36]). On the other hand, if we think of our stochastic process as coming from a Markov chain or, more generally, a dynamical system, then there is an equivalence between EVL and Hitting Time Statistics (HTS) (see [21]), which then yields a significant body of literature coming from that side on these error terms [25,1,2,7,5,31]. In this paper, taking inspiration from all these areas, we obtain sophisticated estimates on rates of convergence, where the dependence on time and 'length' (i.e., the distance from the maximum) scales is made explicit. We will first give general error terms under very general mixing conditions (in a way that unifies both clustering and non-clustering cases), and then impose some stronger conditions on our underlying process to obtain better estimates.

1.1. A more technical introduction

Let $X_0, X_1, \ldots$ be a stationary stochastic process, where each random variable (r.v.) $X_i : \mathcal{Y} \to \mathbb{R}$ is defined on the measure space $(\mathcal{Y}, \mathcal{B}, \mathbb{P})$. We assume, without loss of generality, that $\mathcal{Y}$ is a sequence space with a natural product structure so that each possible realisation of the stochastic process corresponds to a unique element of $\mathcal{Y}$ and there exists a measurable map $T : \mathcal{Y} \to \mathcal{Y}$, the time evolution map, which can be seen as the passage of one unit of time, so that

$$X_{i-1} \circ T = X_i, \quad \text{for all } i \in \mathbb{N}.$$

Note. There is an obvious relation between T and the *shift* map but we avoid that comparison here because we are definitely not reduced to the usual shift dynamics, in the sense that normally the shift map acts on sequences from a finite or countable alphabet, while here T, acts on spaces like $\mathbb{R}^{\mathbb{N}}$, in the sense that the sequences can be thought as being obtained from an alphabet like $\mathbb{R}$.

Stationarity means that $\mathbb{P}$ is T-invariant. Note that $X_i = X_0 \circ T^i$, for all $i \in \mathbb{N}_0$, where T^i denotes the i-fold composition of T, with the convention that T^0 denotes the identity map on $\mathcal{Y}$.

We denote by F the cumulative distribution function (d.f.) of X_0, *i.e.*, $F(x) = \mathbb{P}(X_0 \le x)$. Given any d.f. F, let $\bar{F} = 1 - F$ and let u_F denote the right endpoint of the d.f. F, *i.e.*, $u_F = \sup\{x : F(x) < 1\}$. We say we have an *exceedance* of the threshold $u < u_F$ at time $j \in \mathbb{N}_0$ whenever $\{X_j > u\}$ occurs.