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Some distributivity-like results in the medieval arithmetic of Jordanus Nemorarius and Campanus de Novara

Leo Corry

Cohn Institute for History and Philosophy of Science, Tel Aviv University, 69978, Israel

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Abstract

The present article explores the treatment of some distributivity-like properties in the works of Jordanus Nemorarius and Campanus de Novara. The perspective afforded by this analysis gives rise to some interesting insights concerning medieval attitudes towards the relationship between geometry and arithmetic, in particular as part of the Euclidean tradition. It also sheds interesting light on medieval conceptions about arithmetic as an autonomous discipline requiring its own proper foundation.

Resumen

El presente artículo discute la forma en que propiedades quasi-distributivas de las operaciones aritméticas son presentadas en las obras de Jordanus Nemorarius y Campanus de Novara. La perspectiva ofrecida por este análisis trae a luz algunas observaciones interesantes sobre las actitudes medievales concernientes a la relación entre la geometría y la aritmética, y en particular en lo que corresponde a la tradición euclidiana. La discusión también arroja luz sobre las concepciones medievales de la aritmética como disciplina autónoma que requiere una fundamentación propia y adecuada.

MSC: 01A35

Keywords: Euclid's Elements; Medieval arithmetic; Jordanus; Campanus

1. Introduction

This article explores some "distributivity-like" properties of operations on natural numbers as they surface in the texts of two of the most important mathematicians active in the 13th century, Jordanus Nemorarius and of Campanus de Novara. More specifically it focuses on their role among the basic propositions that these two authors took to be fundamental for their pioneering attempts at providing systematic

E-mail address: corry@post.tau.ac.il.

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foundations for the corpus of arithmetic knowledge. The main point of interest in this topic is that it sheds light on a peculiar aspect of the processes of border-blurring across the domains of continuous and discrete magnitudes in the medieval Euclidean tradition. These domains were strictly kept apart in the original Euclidean text, but the changing contexts where this text was translated, commented, and relied upon—from late antiquity, via the Islamic tradition, and into the medieval versions—brought about the gradual incorporation of new ideas and led to innovative ways to understand and further develop its mathematical contents. One aspect of these processes is manifest in the way that arithmetic and proto-algebraic ideas became gradually associated with what was once a purely geometrical conception of Book II (Corry, 2013). Another aspect, closely related to it, is that of the ideas related with the "distributivity-like" properties that I want to discuss here.

The article opens with a brief overview on the way that "distributivity-like" results appear in different contexts in the *Elements*. Next it discusses the transformations undergone by these results in al-Nayrīzī's commentary to the *Elements*, dating from the early tenth century. This commentary had considerable impact on medieval authors such as Jordanus and Campanus, whose works are discussed in the last two sections of the article. In a separate article (Corry, 2016), I discuss in greater detail similar, "distributivity-like" results as they appear in works not analyzed here, including all the relevant results appearing in Books II, V and VII of Euclid's *Elements*, and then in works such as those of Abu Kāmil, *Liber Mahameleth*, Fibonacci and Gersonides.

It is relevant to stress that my focus on "distributivity-like" properties is not meant to imply that, in any of the texts discussed here, we find a general, clearly formulated idea of "distributivity" as a fundamental, widely acknowledged, general kind of property underlying the relationship between two basic and also well-defined operations, "product" and "addition". Rather, the ideas discussed here in relation with "distributivity-like" properties developed and consolidated separately, though in parallel and in interaction with each other, as part of a long historical process: product, addition, number, magnitudes and also distributivity. I think that it is historically rewarding to look at them from a common perspective that involves a broad idea of "distributive-like" properties. Accordingly, then, the term is used here as a somewhat loose label that allows a common reference to various kinds of results, rather than as an assertion that this was a clearly conceived, general idea specifically applied in particular cases.

2. Distributivity-like properties in Euclid's *Elements*

I start with a brief overview of how some distributivity-like properties appear in Euclid's *Elements*, and in the first place, in Book II. The first four propositions of this part of the treatise discuss some basic properties of area-formation in rectangles. Proposition II.1, for instance, formulates the following general property[1]:

> **II.1:** If there are two straight lines, and one of them is cut into any number of segments whatever, then the rectangle contained by the two straight lines equals the sum of the rectangles contained by the uncut straight line and each of the segments.

The main step in the proof relies on a proposition from Book I, I.34, while making reference to the following diagram, where in the rectangle *BGHC*, the side *BG* equals the given line *A*, and the line *BC* is divided into sub-segments (see Figure 1).

The said proposition I.34 is used to assert that, since *BK* is by construction a rectangle, then *DK* equals *BG* and hence equals *A*. A repeated application of this argument allows concatenating the three resulting rectangles into a single, larger one, and thus to complete the proof.

[1] All the quotations of the *Elements*, including the accompanying diagrams, are taken from Heath (1956 [1908]).