Different cultures of computation in seventh century China from the viewpoint of square root extraction

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Abstract

The aim of this paper is to bring to light a previously unknown geometrical method for extracting the square root in seventh century China. In order to achieve this goal, a seventh century commentary by the scholar Jia Gongyan, 賈公彥, on a Confucian canon, the *Rites of Zhou Dynasty* [*Zhouli* 周禮], is analysed. This is compared with the commentary by his contemporary Li Chunfeng, 李淳風, which is referred to in another mathematical book, the *Mathematical Procedures of the Five Canons*, [*Wujing Suanshu* 五經筭術]. Although these two scholars probably knew each other, they used very different methods to solve the same problem in relation to square root extraction. It is argued that the differences mainly lie in two aspects: firstly, Jia Gongyan mostly made use of geometry while Li Chunfeng used counting rods; secondly, the two methods had different geometrical interpretations. Given the fact that the method of square root extraction Jia Gongyan uses is one among many other methods he employed in mathematics, and it has the same features as the others; moreover, other commentators on the Confucian Canons use similar mathematical methods, this paper closes with a general discussion on mathematical cultures. It is suggested that there were three elements to mathematical practice in seventh century China: geometry, counting rods, and written texts. The interplay and structure between the three elements is seen to influence mathematical practices.

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摘要

　　本文通過分析唐代學者賈公彥對《周禮 ●考工記》 "參分弓長" 和 "軹前十尺" 的注疏，揭示出一種先前未被學術界關注到的幾何開方術。繼而，將之與李淳風對《五經筭術》中 "參分弓長" 問題的註解相比較，我們可以看到：儘管賈公彥與李淳風很可能互相認識，但是他們用了截然不同的開方辦法去處理同樣的問題。本文論證兩者的差別主要在兩方面：一，賈公彥的算法是利用幾何，而李淳風的算法是利用筭籌；二，兩種算法具有不同的幾何解釋。基於如下事實：賈公彥開方術的特色在他其他的注疏中也有體現，而且唐代其他學者對於儒家經典的注疏中也有類似算法，本文就唐代數學文化做一般性的討論，認為：唐代的數學實作主要包括三大要素：幾何，筭籌和文本，三者之間的互動及其形成的結構對唐代數學文化產生了重要影響。

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1. Introduction

The *Nine Chapters on Mathematical Procedures* [*Jiuzhang Suanshu* 九章筭術, hereafter the *Nine Chapters*] includes a procedure, called Kai Fang Shu[1] [開方術]. This procedure, using counting rods, is very significant because it presents the first complete method in the history of mathematics in China for what in modern mathematics is known as square root extraction.[2] Historians have published many articles,[3] not only on this procedure in the *Nine Chapters*, but also on other procedures for root extraction, including square roots and cube roots, in the *Ten Classic Books of Mathematics*, [*Suanjing Shishu* 筭經十書 hereafter the *Ten Mathematical Books*].[4] Thanks to this meticulous work, it is known that despite minor differences, all the procedures in these mathematical canons, written by different scholars, have the same basic features.[5] The various procedures in the *Ten Mathematical Books* share the same general characteristics[6]: the instruments, counting rods,[7] used for carrying out the procedures are the same; the terminology used is very similar; and, there is a common geometrical basis underlying the procedures, which was first presented by Liu Hui[8] 刘徽 in the third century, and was subsequently confirmed by Li Chunfeng[9] 李淳風 in the seventh century.[10]

However, this topic is the subject of much debate and some unsolved problems remain. For example, the details of the procedures carried out with counting rods, and the minor differences between the procedures.

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[1] This procedure is to extract the square root in modern mathematics. Literally speaking, Kai means to open/setup, Fang means square, and Shu means procedure. Therefore, the literal interpretation will be to open/set up a square.

[2] In the 19th century, after mathematics was introduced from the west into China, Qing scholars borrowed the ancient term "Kai Fang" to designate the operation of square root extraction.

[3] All works are divided into two categories: books in which scholars discussed this issue in one or more chapters, and independent papers on this issue. I just mention some of them here: Qian Baocong, 1964, 46–51; Li Yan, 1937, 63–68; Li Jimin, 1990, 389–396; Guo Shuchun, 1992, 29–33; Liu Dun, 1993, 191–198; Joseph Needham, 1959, 62–67; Karine Chemla, 1994; Jean-Claude Martzloff, 2006, 221–228.

[4] During the seventh century, the *Ten Classic Books of Mathematics* includes: the *Mathematical Canon on Gnomon of Zhou* [*Zhoubi Suanjing* 周髀筭經], the *Nine Chapters on Mathematical Procedures* [*Jiuzhang Suanshu* 九章筭術], the *Mathematical Canon on Sea Island* [*Haidao Suanjing* 海島筭經], the *Mathematical Canon of Sunzi* [*Sunzi Suanjing* 孫子筭經], the *Mathematical Canon of Zhang Qiujian* [*Zhang Qiujian Suanjing* 張丘建筭經], the *Mathematical Canon of Xia Houyang* [*Xia Houyang Suanjing* 夏侯陽筭經], the *Joint Mathematical Procedures* [*Zhuishu* 綴術], the *Mathematical Canon on Five Bureaus* [*Wucao Suanjing* 五曹筭經], the *Mathematical Procedures of the Five Canons* [*Wujing Suanshu* 五經筭術], and the *Mathematical Canon on Continuation of Ancient Mathematics* [*Jigu Suanjing* 緝古筭經]. In 263 CE, Liu Hui commented on the *Nine Chapters on Mathematical Procedures*; and in 656 CE, Li Chunfeng and his colleagues edited, commented and made canonical versions for these ten mathematical books, which were used in Imperial University [國子監]. Later, when the book of the *Joint Mathematical Procedures* was lost, Bao Hanzhi [瀚鮑之 13th century] replaced it by another mathematical book that was the *Notes on Traditions of Numerological Procedures* [*Shushu Jiyi* 數術記遺]. 1963, Qian Baocong produced critical editions for these books. Guo Shuchun and Liu Dun produced critical editions recently [1998].

[5] Here what I stress is that most scholars agree to call the procedures "square roots extraction using counting rods" [籌筭開方術], although the procedures evolved over time.

[6] Jean-Claude Martzloff, 2006, 222.

[7] The use of counting rods for square root extraction lasted until the 16th century and the rise in adoption of the abacus.

[8] Liu Hui is the earliest known commentator of the *Nine Chapters* that was handed down through the written tradition. His commentary on the *Nine Chapters* dates from 263 CE.

[9] Li Chunfeng was a 7th century scholar. He and his colleagues edited, and commented on the *Nine Chapters* in 656 CE.

[10] Liu Hui first explained the geometrical basis. The fact that "Fang" in the term "Kai Fang" means square also implies its correlation to geometry.