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Tactics: In search of a long-term mathematical project (1844–1896)

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Abstract

This paper tackles the history of tactics, a field of investigation at the crossroads of algebra, combinatorics and recreational mathematics. Tactics was only taken up by mathematicians now and then between the 1850s and the 1900s, and its emergence was a process of mathematization of questions linked to the notions of "order" and "position". To understand the long-term history of this field of investigation—one that became neither a theory nor a discipline—the paper analyzes the different historical configurations in which tactics took on its scientific meaning. It thus investigates how, under the banner of tactics, a continuity could be claimed by mathematicians that were, finally, working in very different scientific and historical context.

Résumé

Cet article aborde l'histoire de la tactique, un domaine de recherche au croisement de l'algèbre, des combinatoires et des récréations mathématiques. Entre les années 1850 et les années 1900, la tactique n'a été étudiée que de manière épisodique par certains mathématiciens, qui cherchaient à mathématiser les notions d'"ordre" et de "position". Pour comprendre dans la longue durée l'histoire de ce thème de recherche, qui n'est finalement devenu ni une théorie ni une discipline, cet article propose d'analyser les différentes configurations historiques dans lesquelles les travaux portant sur la tactique ont pris sens. Il s'agit donc de comprendre comment, sous la bannière de la tactique, des mathématiciens ont pu se réclamer d'une continuité avec des travaux antérieurs, tout en travaillant dans un contexte scientifique et historique radicalement différent.

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1. Two mathematical papers about Tactics thirty-two years apart

In 1896, the American mathematician Eliakim H. Moore (1862–1932) published a paper in the *American Journals of Mathematics*, the title of which—"Tactical Memoranda"—suggested that tactics was a mathematical field[1] that should be remembered by mathematicians [Moore, 1896]. However, the word "tactics" was far from being frequent in the mathematical literature. In order to introduce tactics, Moore had to reference an old paper by Arthur Cayley (1821–1895) entitled "On the Notion and Boundaries of Algebra" [Cayley, 1864], where Cayley defined tactics as one of the "two great divisions of Algebra":

> Algebra is an Art and a science; *quà* Art, it defines and prescribes operations that are either tactical or logistical; viz. a tactical operation is one relating to the arrangement in many manners of a set of things; a logistical operation [...] is the actual performance, as to obtain for the result a number of any arithmetical operations.

According to Cayley, tactics related to the arrangement of sets of things, whereas logistics (which he took to be synonymous with arithmetic) related to the calculation over numbers using the four algebraic operations. Hence, tactics was much more than merely a peculiar mathematical theory; it belonged to the general principles according to which the algebraic art worked. It could even be separated off "altogether from Algebra", to make "a different branch of mathematical Science". However, with this definition, Cayley put on an equal footing two fields that were far from being at the same stage of theoretical development. Whereas arithmetic (the logistic part of Algebra) was well established and well known, tactics was quite a marginal field at the time. Moreover, in this paper at least, Cayley's purpose remained mainly epistemological—he did not say more about the kind of mathematics one could do with tactics, or about the mathematical objects with which it dealt precisely.

The reference to Cayley in the first line of his article notwithstanding, Moore neither concentrated his attention on this epistemological issue nor on a general definition of tactics. Far from taking a top down and synthetic viewpoint, as Cayley did, he constructed his article as a true mathematical patchwork. In fact, Moore described tactics through the objects with which it dealt—"tactical configurations" and "tactical systems"—whose origins lay in geometrical research done at the end of the 19th century [Reye, 1882; De Vries, 1894] as well as in recreational mathematics [Power, 1867; Sylvester, 1893]. Moore's treatment of these tactical objects was group-theoretic, using in particular the research of the French mathematicians Jordan and Mathieu [Jordan, 1872; Mathieu, 1860, 1861]. Finally, the last quarter of the article offered, as an application, a mathematical treatment of the purpose developed in a book about the card game, Whist [Mitchell, 1891].

Through these two papers, tactics appears as a set of mathematical practices of arranging, combining and ordering sets of objects that were considered essential to a large part of algebraic activity, but that stood at the crossroads of several mathematical fields and topics—algebra and geometry, combinatorics and group theory and, finally, recreational mathematics. Nevertheless, the papers of Cayley and Moore are only two tips of the tactical iceberg. In fact, the first actual occurrence of the word "tactics" in mathematics can be found in a footnote to a lecture that James Joseph Sylvester read in 1854 [Sylvester, 1854, 5] and, as we will see, tactics was taken up now and then between the 1860s and the 1930s. However, none of the papers that mentioned it gave a proper definition or a set of propositions and theorems that would at least delimit its scope precisely. It does not seem to have been taught in any mathematical class either, and no monograph has ever been written about it. How can we explain that such a hybrid collection of "more or less connected topics of Tactic" [Moore, 1896, 264] continued to attract interest over this period? If several

[1] I am using the word "field" in a general sense; it is not intended to be a translation of the concept of "champ" as used by the sociologist Pierre Bourdieu.