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On the origins and foundations of Laplacian determinism

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ABSTRACT

In this paper I examine the foundations of Laplace's famous statement of determinism in 1814, and argue that rather than derived from his mechanics, this statement is based on general philosophical principles, namely the principle of sufficient reason and the law of continuity. It is usually supposed that Laplace's statement is based on the fact that each system in classical mechanics has an equation of motion which has a unique solution. But Laplace never proved this result, and in fact he could not have proven it, since it depends on a theorem about uniqueness of solutions to differential equations that was only developed later on. I show that the idea that is at the basis of Laplace's determinism was in fact widespread in enlightenment France, and is ultimately based on a re-interpretation of Leibnizian metaphysics, specifically the principle of sufficient reason and the law of continuity. Since the law of continuity also lies at the basis of the application of differential calculus in physics, one can say that Laplace's determinism and the idea that systems in physics can be described by differential equations with unique solutions have a common foundation.

1. Laplace's statement of determinism

Histories of determinism in science usually start with the following quote, from Laplace's *Essai philosophique sur les probabilités* (1814):

> An intelligence which, for one given instant, would know all the forces by which nature is animated and the respective situation of the entities which compose it, if besides it were sufficiently vast to submit all these data to mathematical analysis, would encompass in the same formula the movements of the largest bodies in the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes (Laplace, 1814, pp. 3–4).[1]

Throughout history, there have been many people arguing for the general idea that everything that happens is necessary or predetermined. The importance of this quote by Laplace lies in the fact that it is not merely an expression of everything in nature being fixed and everything that occurs being necessary, but that it states that prediction is possible through mathematical analysis, on the basis of the forces and the "respective situations of the entities" that are present. It seems that Laplace is stating a fact about physics, namely that the fundamental equations of physics can be solved in principle (although not necessarily in practice) and then give a unique prediction of future states. This suggests that everything is fixed according to *laws of physics* of a mathematical form, and that it is physics that tells us that the world is deterministic. (Laplace, by the way, did not use the word "determinism", which only received its current meaning later on; see Hacking (1983)).

An interpretation of what Laplace had in mind would then be the theorem that for all systems in classical physics, there are equations of motion of the form

$$\frac{d^2r}{dt^2} = F(r) \tag{1}$$

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[1] "Une intelligence qui pour un instant donné, connaîtrait toutes les forces dont la nature est animée, et la situation respective des êtres qui la composent, si d'ailleurs elle était assez vaste pour soumettre ces données à l'analyse, embrasserait dans la même formule, les mouvements des plus grands corps de l'univers et ceux du plus léger atome: rien ne serait incertain pour elle, et l'avenir comme le passé, serait présent a ses yeux."

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with r the position of a body and $F(r)$ the force to which it is subjected, and that these equations have a unique solution for given initial conditions $r(t_0) = r_0$ and $\frac{dr}{dt}(t_0) = v_0$. This means that if we know the positions and velocities of all particles at a certain instant, and all the forces that are present, we can uniquely determine the future (as well as past) states of the system. This is nowadays the canonical formulation of determinism in classical physics (see e.g., Arnold, 1989, pp. 4–8; Landau & Lifshitz, 1976, pp. 1–2).[2] Whether it holds is a different issue: in particular Earman (1986) and Norton (2008) have shown that there are various cases in which this determinism breaks down. But it is usually supposed that this is what Laplace had in mind when he wrote the above statement.

In this paper I examine in how far this interpretation of Laplace's statement holds, and put forward an alternative interpretation, according to which Laplace's determinism is based on general principles, rather than derived from the properties of the equations of mechanics: specifically, it is based on the principle of sufficient reason and the law of continuity. This does not mean that Laplace's determinism is unrelated to developments in physics, but the relation is not as straightforward as one may suppose.

In Section 2, I discuss the possible foundations of Laplace's deterministic statement in his mechanics. I show that Laplace could not have proven that the equations of motion in mechanical systems always have a unique solution, since this depends on a theorem about uniqueness of solutions to differential equations that was only developed later on (furthermore, the theorem left a possibility for indeterminism open). In Section 3, I discuss the philosophical argument that Laplace provides for his determinism: in fact, the only motivation that he explicitly gives for his determinism is Leibniz' principle of sufficient reason. In Section 4, I show that there was a strong eighteenth century background to Laplace's ideas: he was far from the first to argue for determinism, and also far from the first to do so in terms of an intelligence with perfect knowledge and calculating capacities. Tracing out these connections makes it possible, in Section 5, to get a better understanding of the foundation of Laplace's determinism and the relation with the possibility of mathematical descriptions of nature.

2. The physical foundations of Laplace's determinism

One reason to question the interpretation that Laplace's determinism is based on the idea that there are equations of motion for mechanical systems which always have a unique solution for given initial conditions, is that he never argues for this explicitly; he never makes the argument more explicit than in the above quote. And in this quote, the argument is not formulated very carefully: he does not explicitly say that the equations of mechanics always have a unique solution for given initial conditions, and if this is what he intended to argue, then it is striking that he fails to mention that besides the initial positions of all bodies, one also needs to know their initial velocities in order to solve the equations. Furthermore, from the context of this quote, it appears that he was not trying to say something about the properties of the theory of mechanics. Rather, he was arguing for the importance of probability theory: the above statement appeared in the preface of a work on probability theory, in which Laplace argues that the use of probability theory is not restricted to cases in which there is fundamental chance or randomness. He argues that with perfect knowledge of the present state of the universe, it should be possible to predict future states with certainty; however, in all cases in which our knowledge is less than perfect, we have to rely on probability theory. Thus, it is because of our ignorance that we have to take recourse to probability theory, even though at the bottom of things, there is a solid necessity.[3]

The idea that certain prediction should be possible on the basis of perfect knowledge of the present state of the universe is something that Laplace had considered before. While his 1814 quote has become famous, he already made a similar statement many years earlier, in a lecture in 1773 (printed in 1776), at the very beginning of his career. The statement is made in a similar context, to clarify the notions of necessity and probability, but it is formulated in less physical terms and does not refer to mathematical analysis or calculation, and it is less apparent that it has a foundation in physics:

> ...if we conceive an intelligence which, for a given instant, encompasses all the relations between the beings of this universe, it may determine, for any time in the past or the future, the respective position, the motions, and generally the affections of all those beings (Laplace, 1773, p. 144).[4]

Despite the fact that Laplace did not explicitly demonstrate that the equations of motion always have a unique solution, one may want to argue that he simply knew that this was the case; one may even want to argue that this was so obvious to him that he did not think further demonstration was needed. However, it is not that easy to show that the equations of motion always have a unique solution. In fact, such a demonstration depends on a theorem about the existence and uniqueness to differential equations that was only developed later on: therefore, Laplace could have no proof available.

The first person to work on the issue of existence and uniqueness of solutions to differential equations was Cauchy, who showed in the 1820's that an equation like [1] has a unique solution if $F(r)$ is continuously differentiable (Kline, 1972, p. 717). In 1876, Lipschitz gave a more precise analysis, by showing that the function $F(r)$ does not necessarily have to be continuously differentiable in order for an equation like [1] to have a unique solution; it is enough if it fulfils the condition that there is a constant $K > 0$ such that for all r_1 and r_2 in the domain of F,

$$|F(r_1) - F(r_2)| \leq K|r_1 - r_2|.$$

(Lipschitz, 1876). This condition came to be known as 'Lipschitz continuity'.

These mathematical results imply that the equations of motion of a classical system can fail to have a unique solution if they involve a force which is not Lipschitz continuous. This fact has recently been used by Norton (2008) to argue that determinism can fail in classical mechanics; but it was already noted a couple of times before, most notably by Boussinesq in (1879) (see Van Strien, in press). In (1806), several years before Laplace's famous statement of determinism in 1814, Poisson already discussed the possibility that the equations of motion of a system in physics fail to have a unique solution. One of the cases he discussed was that of a body subjected to a force $F(r) = cr^a$ with c and a constants, and

[2] One can make a distinction, however, between determinism and predictability: the equation of motion plus initial conditions may uniquely determine future states of the system, but that does not necessarily mean that we can calculate these future states (see Earman, 1986, p. 7). Laplace did not make this distinction; rather, he made a distinction between predictability in principle and in practice.

[3] As Daston (1992) shows, earlier authors on probability theory such as Jakob Bernoulli and De Moivre also argued for determinism, with a similar aim (but in different terms).

[4] "...si nous concevons une intelligence qui, pour un instant donné, embrasse tous les rapports des êtres de cet Univers, elle pourra déterminer pour un temps quelconque pris dans le passé ou dans l'avenir la position respective, les mouvements, et généralement les affections de tous ces êtres." The lecture is very mathematical; this statement is placed right in the middle. According to Hahn (2005), this shows that Laplace did not assign an important role to metaphysics and that he wanted to be foremost regarded as a mathematical physicist (Hahn, 2005, p. 53). Hahn also points out the use of the word êtres which suggests that it applies to all beings (also living ones).