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Short communication

A two-dimensional nonlinear model for the generation of stable cavitation bubbles

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A B S T R A C T

Bubbles appear by acoustic cavitation in a liquid when rarefaction pressures attain a specific threshold value in a liquid. Once they are created, the stable cavitation bubbles oscillate nonlinearly and affect the ultrasonic field. Here we present a model developed for the study of bubble generation in a liquid contained in a two-dimensional cavity in which a standing ultrasonic field is established. The model considers dissipation and dispersion due to the bubbles. It also assumes that both the ultrasonic field and the bubble oscillations are nonlinear. The numerical experiments predict where the bubbles are generated from a population of nuclei distributed in the liquid and show how they affect the ultrasonic field.

1. Introduction

When the amplitude of an ultrasonic wave traveling in a liquid passes over the cavitation threshold of this liquid, nonlinearly oscillating bubbles are generated from a population of very tiny nuclei of gas that are already present in the liquid. When the effect of ultrasound is not violent, these bubbles oscillate nonlinearly around their equilibrium size without collapsing during many acoustic cycles. This process received the name of stable cavitation [1], as opposed to inertial cavitation that is characterized by the generation of unstable oscillating bubbles that grow and collapse. Several industrial activities based on cavitation bubbles exist [2–7]. Sonochemistry relies on the behavior of cavitation bubbles under the effect of ultrasound [8]: huge temperatures reached in collapsing bubbles provoke chemical reactions. This application of high-power ultrasound is currently the framework of very active basic and applied researches. The efficiency of sonochemistry processes can be assessed by computational simulations [9]. In particular, it is fundamental to understand the mechanism from which nonlinearly vibrating bubbles are generated in an ultrasound field within a sonoreactor. This is the purpose of this paper.

Cavitation is a nonlinear process: bubbles are created at high pressure amplitudes and oscillate nonlinearly, they increase the nonlinearity of the liquid and the ultrasonic field becomes nonlinear [10]. This mechanism was observed and described in seminal papers [11,12]. Several models that assume linear acoustic waves were developed to simulate this phenomenon [13–16]. A specific aspect of these works is that above the threshold the void fraction was considered constant in Ref. [13] and assumed to be linearly dependent on some pressure amplitude values in Refs. [14–16]. A review on cavitation models was recently published by Tudela et al. [17]. Their main conclusion is that the simulation of the cavitation process must be nonlinear. Several works on the formation of different cavitation structures, such as cone-like or streamer structures, have also been performed [18–21]. Special mention is given to the work by An, in which the generation of cavitation clouds in a standing wave is simulated qualitatively [22].

Nonlinear bubble oscillations are generally considered for modeling the propagation of ultrasound in a bubbly liquid. However, a linear pressure field is usually assumed. In this paper we consider nonlinearly oscillating bubbles and nonlinear pressure waves for the modeling of their interactions [23]. The model presented in Ref. [23] simulates these interactions considering existing bubbles in a liquid, but it cannot model the generation of bubbles. We use that two-dimensional model to create a simulation tool for the cavitation process in a sonoreactor through the development of a method based on a nonlinear dependence of the void fraction on rarefaction pressures above a given threshold value. The model predicts the distribution of bubbles and acoustic pressure due to these nonlinear interactions.

2. Model

The model used to predict the ultrasonic field in the bubbly liquid can be found in Ref. [23] (Snow-Bl code). This model simulates the interaction between the gas-bubble oscillations and ultrasound

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by considering both fields as nonlinear. It also accounts for dissipation and dispersion due to the bubbles. It solves a differential system that couples the acoustic pressure p and the bubble volume variation v. Both variables are functions of two-dimensional space coordinates, x and y, and time t. It is written as follows, where ∇^2 is the Laplacian operator, ω_{0g} is the bubble resonance, c_{0l} and ρ_{0l} are the low amplitude sound speed and density at equilibrium of liquid, δ is the viscous damping coefficient of the bubbly liquid, $\eta = 4\pi R_{0g}/\rho_{0l}$, $a = (\gamma_g + 1)\omega_{0g}^2/(2v_{0g})$, $b = 1/(6v_{0g})$, γ_g is the specific heats ratio of gas, R_{0g} and v_{0g} are the initial radius and volume of bubbles:

$$\begin{cases} \partial^2 v/\partial t^2 + \delta\omega_{0g}\partial v/\partial t + \omega_{0g}^2 v + \eta p \\ \quad = av^2 + b(2v\partial^2 v/\partial t^2 + (\partial v/\partial t)^2), 0 \leqslant x \leqslant l_x, 0 \leqslant y \leqslant l_y, 0 < t < t_{max} \\ \nabla^2 p - 1/c_{0l}^2 \partial^2 p/\partial t^2 = -\rho_{0l} N_g \partial^2 v/\partial t^2, \\ \qquad 0 < x < l_x, 0 < y < l_y, 0 < t < t_{max} \\ p(x \neq 0, t = 0) = v(t = 0) = \partial p/\partial t(x \neq 0, t = 0) = \partial v/\partial t(t = 0) = 0, \\ \qquad 0 \leqslant x \leqslant l_x, 0 \leqslant y \leqslant l_y \\ p(x = 0) = p_0 \sin(\omega_f t), p(x = l_x) = 0, 0 \leqslant y \leqslant l_y, 0 < t \leqslant t_{max} \\ p(y = 0) = p(y = l_y) = 0, 0 \leqslant x \leqslant l_x, 0 < t \leqslant t_{max} \end{cases} \quad (1)$$

In this system, l_x and l_y are the dimensions of the cavity, t_{max} is the last instant of the study, $\omega_f = 2\pi f$ is the source pulsation of frequency f and amplitude p_0 used to excite the cavity, $N_g(x, y)$ is the bubble density in the cavity evaluated via the following law (represented in Fig. 1) at the end of each acoustic period $1/f$ for computing the fields during the subsequent period:

$$N_g(x, y) = N_{g\,min} + 0.5N_{g\,max}\tanh(0.0004(\tilde{p}(x, y) - \Phi_c - 0.6 \times 10^4)) + 0.4N_{g\,max}, \quad (2)$$

in which $N_{g\,min}$ is the low bubble density at the outset, $N_{g\,max}$ is the maximal bubble density, Φ_c is the cavitation threshold, and $\tilde{p}$ is the minimum negative value reached by the rarefaction pressure over the acoustic period $1/f$. The bubble density in the liquid is thus defined as a function of $\tilde{p}$. It also depends on an assumed preexisting population of gas nuclei modeled by a low density ($N_{g\,min}$) of very tiny bubbles that are distributed homogeneously in the liquid at the outset. It is worth noting that many different functions similar to the one given in Eq. (2) could be used, depending on the conditions of the experiments (liquid, working frequency, etc). This law imposes that no bubble creation is possible below Φ_c.

The procedure followed to model the generation of bubbles is performed in two steps at each acoustic period $1/f$: (i) the nonlinear bubble oscillations and the nonlinear acoustic field are computed in the fast scale by Eq. (1) at each instant of the period, (ii) the bubble creation is computed in the slow scale by Eq. (2) at the end of the period and the bubble density obtained is used during the subsequent acoustic period. Since $\tilde{p}$ is evaluated at the end of each cycle, the bubble density is constant over a single period but is varying from cycle to cycle.

In the model the mechanism for bubble creation is considered to start once the standing wave has been established in the cavity (see Figs. 4 and 5). The size of the created bubbles is assumed to be unique, whereas the bubble density does depend on space and time: $N_g(x, y, \text{period})$. Note that Bjerknes forces, acoustic streaming, buoyancy, bubble collapse and other effects are not considered here.

3. Numerical simulations

The liquid considered in this section is water and the bubble gas is air. The driving frequency is 200 kHz. The population of nuclei present at the outset of the experiment is simulated by bubbles of radius 0.1 microns distributed evenly in the cavity with the density of 1×10^{10} bubbles per m^3. The void fraction of the nuclei is 4.2×10^{-9}%. The cavity containing the liquid with the gas nuclei is set to be resonant at the source frequency with this medium. Its dimensions can be seen in Fig. 3: $l_x = 8$ mm and $l_y = 3.9$ mm. When low amplitudes are used at the source in the model the cavitation threshold set at 8 kPa is never reached, the bubble population remains the same, and the acoustic field remains linear. However, when higher amplitudes are used and the cavitation threshold is reached, bubbles of resonance at 0.75 MHz, radius of 4.5 microns, are generated from the nuclei at the density that obeys the law shown in Fig. 1. The highest void fraction that may be obtained with this law is 0.0038%. These bigger bubbles at higher density modify the acoustic properties of the medium. Fig. 2 shows the sound speed (a), attenuation coefficient (b), and compressibility (c) vs. frequency in the regions of highest possible density of biggest bubbles obtained from a perturbative technique applied to the differential system [10]. The dispersive character of the bubbly liquid is clearly seen in the diagrams. These changes vs. frequency affect the ultrasonic field. They actually affect each of the frequency components of the ultrasonic field in a different way. It can be seen that the driven frequency (200 kHz), situated

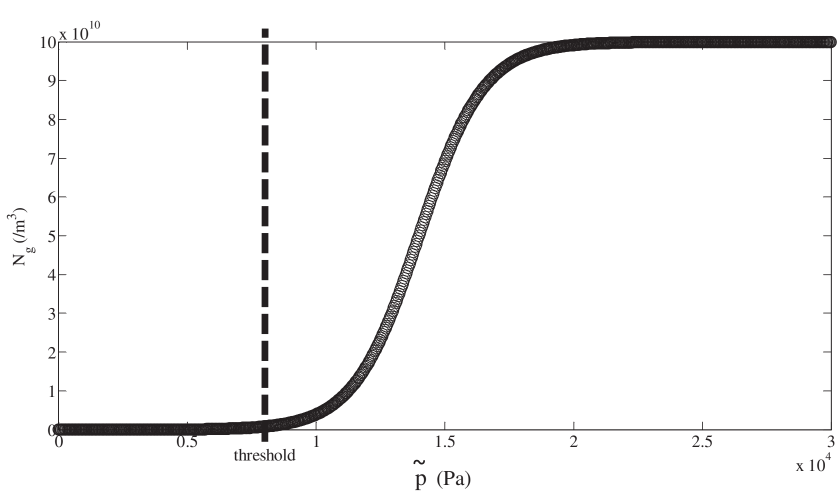

Fig. 1. Bubble density defined as a function of the minimum rarefaction pressure peak over an acoustic cycle.