Ultrasonics Sonochemistry 29 (2016) 524–527

Contents lists available at ScienceDirect

Ultrasonics Sonochemistry

journal homepage: www.elsevier.com/locate/ultson

Mechanics of collapsing cavitation bubbles

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ARTICLE INFO

Article history:
Received 23 September 2014
Received in revised form 17 February 2015
Accepted 6 April 2015
Available online 8 April 2015

Keywords:
Cavitation
Bubbles
Microjets

ABSTRACT

A brief survey is given of the dynamical phenomena accompanying the collapse of cavitation bubbles. The discussion includes shock waves, microjets and the various ways in which collapsing bubbles produce damage.

1. Bubble growth and collapse

In this special issue "Cleaning with bubbles" I will discuss the mechanical aspects of gas bubble implosions in liquids. They are essential for the subject "cleaning with bubbles" but have been studied in the past mainly in connection with cavitation in the flow along the blades of ship propellers and with damage caused by their implosion. A bubble can be produced in many ways: by vaporous growth due to low ambient pressure, as in propeller cavitation, by low pressures due to viscous stresses as occurs in the synovial liquid in your knees when you rapidly stretch these, by introducing locally high energy in the liquid, for example with a laser pulse, or by strong acoustical beams as in machines that pulverize kidney stones. In all these cases bubbles grow first, usually from microscopic size, and finally collapse due to the return of high pressure in their vicinity. The cleaning effect is coming from the final stage of the collapse and therefore it is worthwhile to go into this somewhat further. The earliest studies were undoubtedly in the field of cavitation on ship propellers and we will deal with that example. Rayleigh's [1] mathematical formulation of the collapse of a cavity is

$$R\ddot{R} + \frac{3}{2}(\dot{R})^2 = \frac{p(R) - p(\infty)}{\rho} \quad (1)$$

In this equation R is the radius of the bubble, ρ the density of the liquid and p the liquid pressure, $p(R)$ at the interface and $p(\infty)$ far away from the bubble. A dot means differentiation with respect to time. With initial radius R_0 this can, for an empty cavity, $p(R) = 0$, be integrated to give

$$\dot{R}^2 = \frac{2p(\infty)}{3\rho}\left(\frac{R_0^3}{R^3} - 1\right) \quad (2)$$

In the final stage of the collapse R_0 is much larger than R and the bubble wall velocity u reaches the value

$$u = \left\{\frac{2p(\infty)}{3\rho}\left(\frac{R_0}{R}\right)^3\right\}^{\frac{1}{2}} \quad (3)$$

Liquid rushes into the cavity and with an empty cavity there is a singularity at the end. In reality the bubble contains gas, air mainly, and vapor. These are compressed, condensation may take place at the same time, and the collapse stops finally, at a radius of the bubble so small, that pressures of thousands of atmospheres could be reached provided the bubble stays spherical. To give an idea, suppose a bubble starts to collapse under a pressure difference of one bar far away and a pressure of 0.01 bar internal gas pressure at an initial radius of 1 mm. With a polytropic constant (see below) of 1.4, a minimum radius is reached of 1 μ and the final pressure is about $3 \cdot 10^7$ N/m^2. The time that it takes to reach the minimum radius is of order of 10^{-4} s, using (3) as an estimate. Since Rayleigh [1] many investigations have been conducted to take into account the various physical phenomena that affect the growth and collapse of spherical bubbles. Good accounts are given in the review papers by Plesset and Prosperetti [2], and by Blake and Gibson [3]. There are in addition several books describing the fundamentals of bubble cavitation, such as Young [4], Brennen [5].

Most developments took place in the second half of the twentieth century. Instead of (1) is nowadays used an equation called the Rayleigh-Plesset equation, which takes account of surface tension, viscous stresses, gas and vapor content. So, the pressure $p(R)$ is the

http://dx.doi.org/10.1016/j.ultsonch.2015.04.006
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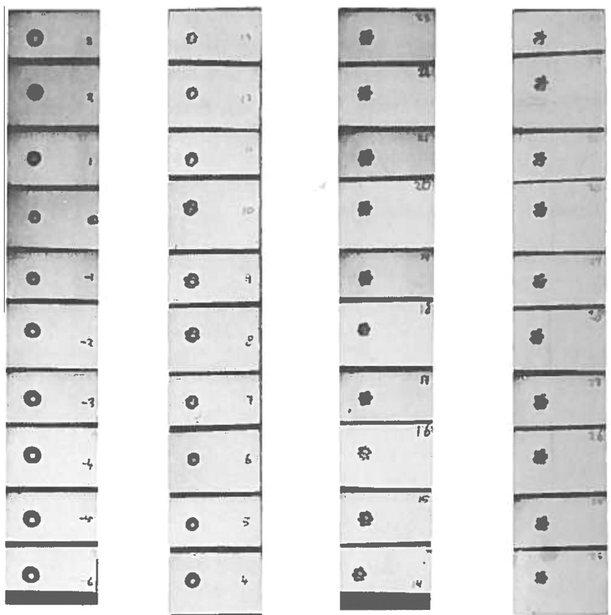

Fig. 1. Collapsing bubble. The initial radius is 1.0 mm. The pressure difference under which the bubble collapses is 0.75 bar. The time difference between successive frames is 40 µs. From [12].

pressure due to the gas content of the bubble, which under compression may behave from isothermal to adiabatic. In the first case the polytropic constant κ in the relation $p(R)/p_0 = (\rho(R)/\rho_0)^\kappa$ is equal to one and in the second case 1.4, for air. In the example above the value 4/3 is used. The Rayleigh–Plesset equation is

$$R\ddot{R} + \frac{3}{2}(\dot{R})^2 = \frac{1}{\rho}\left\{p(R) - p(\infty) - \frac{2\sigma}{R} - \frac{4\mu}{R}\dot{R}\right\} \quad (4)$$

In this equation two properties of the liquid occur apart from the density, the surface tension σ and the dynamic viscosity μ. The terms representing their effect on the collapse are $2\sigma/R$ and $4\mu/R\dot{R}$ respectively. The value of σ for a water-air interface is 0.07 N/m and is therefore only important for very small bubbles. The value of μ is 10^{-3} kg/ms. Viscous dissipation is part of the damping of bubble motions but is in general overruled by dissipation due to heat conduction from the bubbles into the surrounding fluid and acoustic radiation. Therefore the dynamic Eq. (4) is not sufficient to determine gas pressure and other properties of the collapse, but should be accompanied by an energy equation. In the early days of bubble research it was assumed that the gas inside bubbles behaves either isothermally or adiabatically, depending on the thickness of the thermal boundary layer inside the bubble as compared to the radius. This leads to only very crude approximations. To obtain reliable predictions, the full set of dynamic and energy equations should be applied. Analytic solution is then in general not possible, and numerical analysis is needed. To clarify the complex connection between thermodynamic and dynamic behavior of bubble growth and collapse important contributions have been made by Prosperetti and co-workers see e.g. Watanabe and Prosperetti [6]. High gas pressures as mentioned as an numerical example in connection with Eq. (4) are not fully reached in practice, because collapsing bubbles do not remain spherical. The collapse is unstable, due to the curvature of the interface. Note that a flat interface is stable when the heavy liquid accelerates in to the lighter one. The bubble surface is deformed while the gas inside is compressed. In Fig. 1 is shown how strongly deformed the bubble surface becomes.

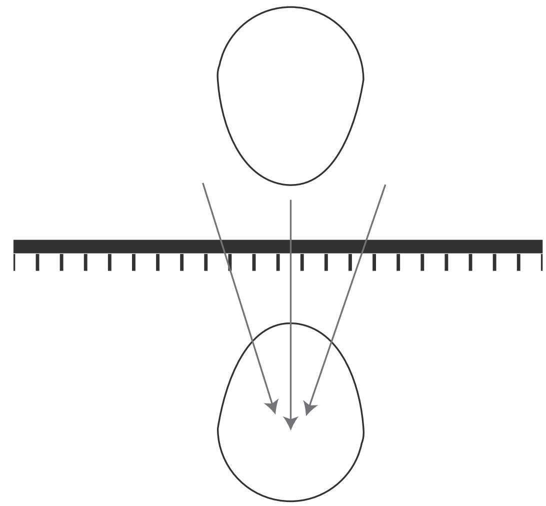

Fig. 3. Bubble collapsing close to a rigid boundary.

In the case of unconfined surroundings there is finally a rebound. Pressure waves are sent out from the bubble in to the ambient liquid. These are so strong that shock waves are formed in the liquid, as shown in Fig. 2.

In this stage of the collapse the compressibility of the liquid becomes important .In the dynamic Eq. (4) the liquid is supposed to be incompressible. Extension of the Eq. (4) to the compressible case then is needed and has been done by several authors, see Lezzi and Prosperetti [8].

2. Bubble collapse close to a boundary

New phenomena occur when the collapse takes place near a boundary. We will concentrate on a boundary which may be considered as rigid. Whether a piece of textile, to be cleaned for

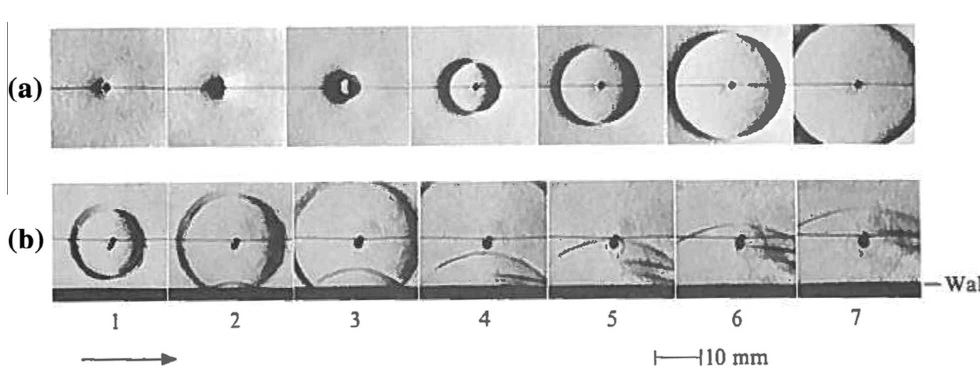

Fig. 2. Shock waves emitted from bubbles. (a) Bubbles far away, compared with their radius, from a boundary, (b) close to the boundary (note the beginning of a microjet in (b). From [7].