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Feedback controller input design for ignition of deuterium–tritium in NSTX tokamak

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ABSTRACT

Nuclear fusion is a nuclear reaction in which two or more atomic nuclei (such as a deuterium–tritium) come very close and then collide at a very high speed and join to form a new high energy nucleus (Helium). Determination of accurate plasma horizontal position during plasma discharge is essential to transport it to a control system based on feedback. The solutions of Grad-Shafranov equation (GSE) analytically can be used for theoretical studies of plasma equilibrium, transport and magneto hydrodynamic stability. Here we have presented specific choices for source functions, kinetic pressure and poloidal plasma current, to be quadratic in poloidal magnetic flux and derive an analytical solution for Grad-Shafranov equation. With applying this solution to NSTX tokamak, we calculated poloidal magnetic flux, toroidal current density and normalized pressure profiles for this tokamak. Toroidal and poloidal flows can considerably change the equilibrium parameters of tokamak. These effects on the equilibrium of tokamak plasmas are numerically investigated using the code FLOW. As a comparative approach to equilibrium problem, the code is used to model equilibrium of NSTX tokamak for case pure toroidal flow. Comparison of the results of these two methods for NSTX tokamak shows good agreement between two and that our analytical solution can be served as good benchmark against the equilibrium code FLOW.

Introduction

For stationary and ideally conducting plasmas, magneto hydrodynamics (MHD) equations plus Maxwell's equations reduce to the two-dimensional, nonlinear, elliptic partial differential equation commonly referred to as the Grad-Shafranov equation (GSE):

$$-\left(\frac{\partial^2\psi}{\partial R^2}-\frac{1}{R}\frac{\partial\psi}{\partial R}+\frac{\partial^2\psi}{\partial Z^2}\right)=\mu_0 R^2 p'(\psi)+FF'(\psi)\equiv\mu_0 R j_\varphi, \tag{1}$$

where j_φ is the toroidal current density, the stream function ψ is the poloidal magnetic flux per radian, $F=RB_\varphi=\frac{\mu_0 I_{pol}(\psi)}{2\pi}$ where I_{pol} is the poloidal current, $p'(\psi)=\frac{dp}{d\psi}$ and $FF'(\psi)=\frac{d\left(\frac{1}{2}F^2\right)}{d\psi}$ are arbitrary functions of ψ. In toroidally confined plasma, the Grad-Shafranov equation, in general, a nonlinear partial differential equation, describes the hydro magnetic equilibrium of the system. The solution of the GS equation provides the magnetic field, the current density, and the kinetic pressure inside axisymmetric plasma in hydro magnetic equilibrium.

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Having analytical solutions to this equation is convenient to configure physical equilibria as a basis for theoretical studies of transport, waves and stability. Also they can be served as good benchmarks for testing numerical MHD equilibrium codes. The GS equation is an elliptic partial differential equation (PDE) for the poloidal magnetic flux ψ that labels the magnetic surfaces in axisymmetric plasma equilibrium. The equation contains two arbitrary functions $p(\psi)$ and $F(\psi)$ that specify the dependence of the kinetic pressure and the poloidal plasma current on the magnetic flux ψ [1]. Many natural phenomena are described by systems of nonlinear PDE's, which are difficult to be solved analytically, as long as a general theory for completely solving nonlinear PDE's is not yet available. The simplest analytical solution to the inhomogeneous GSE is the well known Solov'ev equilibrium [2–10] and corresponds to source functions (*p and F*) linear in Ref. ψ. Equilibrium of this type has been extensively used for equilibrium, transport and stability studies. However the Solov'ev equilibrium solutions are over constrained in shape or in poloidal beta (plasma current). From the same Solov'ev equilibrium case, by expanding the solution of the homogeneous equations in a polynomial form in r (of fourth degree) and z (of second degree) and assuming up/down symmetry, it is possible to describe the plasma shape by four parameters [3]. By using particular source functions for the GSE, it is found a class of exact analytical solutions [4,5,11–20]. Also, it is presented a new family of solutions where the plasma pressure is linear in Refs. ψ, while the squared poloidal current has both, a quadratic and a linear ψ term [21–27]. An exact solution of the large-aspect ratio approximation with an additional assumption of a simple relation between the magnetic flux and the current density was constructed [28–30]. Analytical equilibrium solutions for tokamak plasmas are difficult to find since the equations governing the equilibrium are highly nonlinear. Therefore numerical solutions are always useful. Among those, the FLOW code is developed for the study of axisymmetric tokamak equilibrium in the presence of toroidal and poloidal flow for NSTX tokamak [8]. FLOW was originally designed for spherical tokamaks. It has been repeatedly observed in several devices that when the plasma rotates either toroidally or poloidally, both the energy transport as well as the macroscopic stability improves significantly (the plasma rotation can be either spontaneous or driven by neutral beam injection or radio frequency heating). These effects on the equilibrium of tokamak plasmas are numerically investigated using the code FLOW. FLOW solves the GS-Bernoulli system of equations with a multi grid approach including finite pressure anisotropy. The code input requires the assignment of a set of free functions of the poloidal magnetic flux Ψ, which depend on the so called closure equation governing the temperature(s) or entropy. Though, FLOW can solve the equilibrium equation with arbitrary flow [31–34]. In this paper, in Section Special analytical solution to the GSE we have represented the special analytic solution for GS equation using specific choices for the free functions (*p and F*). It is done with determining the finite number of unknown expansion coefficients in the three term solution. Then we apply the results to NSTX tokamak. In Section Study of equilibrium by FLOW code, as a comparative approach to equilibrium problem for NSTX tokamak, we have showed the results of a numerical study carried out with the equilibrium code FLOW developed to study fixed boundary equilibria with toroidal flows. Following this section, a brief comparison between these two approaches is presented. In Section Study of equilibrium by FLOW code we briefly discuss about results.

Special analytical solution to the GSE

The magnetic field is related to the poloidal flux Ψ by:

$$B_\varphi = \frac{F(\Psi)}{R},\ \boldsymbol{B}_p = \frac{\nabla\Psi \times \boldsymbol{e}_\varphi}{R}, \tag{2}$$

where B_φ and B_p are toroidal and poloidal magnetic fields. We choose the free functions $p(\Psi)$ and $F(\Psi)$ to be quadratic in Ref. Ψ [9]:

$$p = p_{axis}\left(\frac{\Psi^2}{\Psi_{axis}^2}\right),\ F^2 = R_0^2 B_0^2\left[1 + b_{axis}\left(\frac{\Psi^2}{\Psi_{axis}^2}\right)\right] \tag{3}$$

Here Ψ_{axis}, p_{axis} and b_{axis} are constants related to the values of Ψ, p and F on axis and R_0 and B_0 are the major radius and vacuum toroidal field at the geometric center of the plasma. With these choices and specific normalization, the GS equation reduces to:

$$4\varepsilon^2 x\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + (\alpha x + \gamma)\psi = 0. \tag{4}$$

We used these normalizations: $\Psi = \Psi_{axis}\psi$, $R^2 = R_0^2 x,\ z = ay,\ \varepsilon = \frac{a}{R_0},\ \gamma = \left(\frac{aR_0B_0}{\Psi_{axis}}\right)^2 b_{axis},\ \alpha = \left(\frac{aR_0B_0}{\Psi_{axis}}\right)^2 \beta_{axis}$ *and* $\beta_{axis} = \frac{2\mu_0 p_{axis}}{B_0^2}$. The solution of Eq. (4) is found by separation of variables:

$$\psi = \sum_m X_m(\rho)Y_m(y), \tag{5}$$

where $x = \frac{-i\rho\varepsilon}{\sqrt{\alpha}}$. Solution of the Y_m equation for an up/down symmetric configuration (like NSTX) is given by:

$$\frac{d^2Y_m}{dy^2} + k_m^2 Y_m(y) = 0,\ Y_m(y) = \cos(k_m y), \tag{6}$$

where k_m is the mth separation constant, which can be real or imaginary and we should determine it. The $X_m(\rho)$ equation reduces to:

$$\frac{d^2X_m}{d\rho^2} + \left[-\frac{1}{4} + \frac{\lambda_m}{\rho}\right]X_m = 0, \tag{7}$$

with $\lambda_m = -i\frac{\gamma - k_m^2}{4\varepsilon\sqrt{\alpha}}$ and the solutions of $X_m(\rho)$ are Whittaker functions [10]. The solution of $X_m(\rho)$ is given by:

$$X_m(\rho) = Im[a_m W_{\lambda_m,\mu}(\rho) + b_m M_{\lambda_m,\mu}(\rho)]. \tag{8}$$

The a_m and b_m are unknown expansion coefficients that must be determined. Both ρ and λ_m are purely imaginary quantities while X_m must be purely real, then we only keep imaginary parts of Whittaker functions. For our model the Whittaker parameter is $\mu = \frac{1}{2}$. With maintaining three terms in the summation of ψ, we have: