



# Continuous and discrete microstructured materials with null Poisson's ratio



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## ABSTRACT

In this paper we propose different classes of isotropic microstructured media with tunable Poisson's ratio. The elastic periodic systems are continuous porous media and two- and three-dimensional lattices. The microstructural parameters can be tuned in order to have an effective Poisson's ratio equal to zero. The connection between microstructural parameters and effective properties is shown in detail both analytically and numerically.

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## 1. Introduction

The effective behavior of a material depends on the internal structure that is possessed by the material at different scales (see, for example, [1,2]). The capability to design different microstructures can lead to extreme constitutive properties, that cannot be achieved by naturally occurring materials. In the last decades the design of new microstructured media has been accompanied by new technologies in the production of artificial materials, such as 3D printing, 3D laser and multiphoton lithography, with possible advanced applications for ceramic materials, as shown by Bauer et al. [3] and Jang et al. [4].

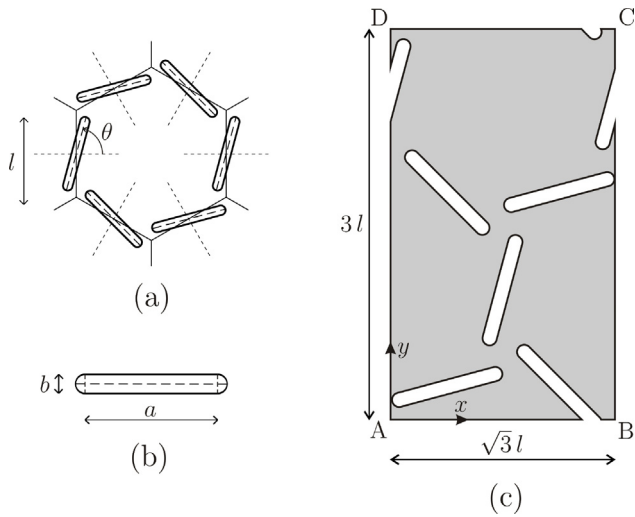
In the present paper we are interested in the design of new microstructures, that can guarantee a Poisson's ratio equal to zero. The purpose of this choice is in the possibility of 'decoupling' the deformation mechanism in different directions, so that when a material is stressed in one direction it does deform only in the direction of the load, but not in the orthogonal directions. Another feature of interest is the design of isotropic materials, so that the 'decoupling deformation' mechanism does not depend on the direction of the application of the load.

For isotropic materials constitutive stability limits the Poisson's ratio  $\nu$  between  $-1$  and  $0.5$ . While negative values of the Poisson's ratio are theoretically possible, most of the naturally occurring materials show positive  $\nu$ . Love [5] mentioned materials with a negative Poisson's ratio, which are named auxetic after Evans [6]. Extended reviews of existing auxetic models can be found in [7–10]. There are very few examples of materials with null Poisson's ratio. A naturally occurring material with a Poisson's ratio close to zero is cork [11], while a three-dimensional spongy graphene and a nanoparticle multilayer have been proposed by Wu et al. [12] and by Nguyen et al. [13], respectively, as artificial systems with  $\nu=0$ . Materials with null Poisson's ratio are very useful for sealing [14] and biomedical applications, such as scaffolds in tissue engineering [15].

Here, we present different classes of microstructured materials: a porous continuum and different classes of lattices. The topology of the microstructures assure an isotropic behavior at least within the linear range of the stress-strain response curve of the material. The effective behavior can be tuned by modulating the microstructural parameters. The microstructures are simple and can be easily produced with existing technologies. The paper is organized as follows. In Section 2 we present the porous medium and we give evidence of the effect of the size and of the relative inclination of the pores on the effective properties. In Section 3 we propose different lattice models, namely a two-dimensional lattice with a hexagonal and a triangular microstructure and a three-dimensional body centered cubic system. In the plane models the effective properties are given

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**Fig. 1.** (a) Hexagonal pattern of the holes in the two-dimensional ceramic structure; (b) illustration of a single hole; (c) representation of the elementary cell of the structure.

analytically, while for the three-dimensional lattice the problem is analyzed numerically. For all the elastic systems analyzed we show the design of isotropic media with Poisson's ratio equal to zero. Final considerations conclude the paper.

## 2. A continuous porous model

We consider a perforated ceramic sheet, which can be designed such that it exhibits a null Poisson's ratio. The holes are disposed in a hexagonal arrangement as sketched in Fig. 1a, where  $\theta$  is the angle measured from the normal to the hexagon side and  $l$  is the hexagon side length. Each hole is made of a rectangle of length  $a$ , ending with two semicircles of diameter  $b$ , as shown in Fig. 1b. The structure is formed by repetitive cells, one of which is drawn in Fig. 1c.

We assume that the matrix is made of magnesium oxide, which is used in several engineering applications for its excellent performance at high temperatures, resistance to corrosion and transparency to infrared light. The matrix is characterised by a Young's modulus  $E_m = 300$  GPa, a Poisson's ratio  $\nu_m = 0.36$  and a yield stress  $\sigma_y = 160$  MPa. The elastic modulus and Poisson's ratio of the homogenised structure, consisting of both the matrix and the holes, will be indicated as  $E$  and  $\nu$ , respectively.

We note that the hexagonal disposition of the holes makes the medium isotropic in the plane (see [16]). As a consequence, the constitutive properties of the perforated sheet can be evaluated by loading or stretching the structure in only one direction.

### 2.1. Finite structure

We start by analysing a perforated sheet of finite dimensions. The structure has a square shape of side  $L = 200$  mm on the plane  $x$ - $y$ , as shown in black color in Fig. 2a, with a thickness  $t = 5$  mm in the  $z$  direction. The dimensions of the microstructure are the following:  $l = 9.0$  mm,  $b = 1.0$  mm,  $a = 0.765l = 6.9$  mm and  $\theta = 75^\circ$ .

We determine the homogenised properties of the perforated sheet by employing a finite element model developed in *Comsol Multiphysics*<sup>®</sup>. We use a mesh of around  $5 \times 10^5$  triangular elements, which is refined near the holes. We impose zero horizontal displacements at the left boundary and apply a horizontal displacement of 0.01 mm on the right boundary.

The deformed configuration is shown in Fig. 2a (in the figure, the scale factor for the displacement is equal to 2000). The colors indicate the values of the von Mises stress, which are detailed on

the right of the figure. We point out that the maximum value of the von Mises stress, detected near the holes, is well below the yield limit  $\sigma_y$  of the matrix. From Fig. 2a it is apparent that the perforated sheet does not contract nor expand laterally as it is stretched, hence it has a null value of the Poisson's ratio.

In order to precisely compute the homogenised Poisson's ratio and Young's modulus of the porous structure, we refer to a square area of side 100 mm in the central part of the model. This area is far enough from the boundaries to neglect the boundary layer effects and it is large enough to contain a sufficient number of elementary cells. In this area, we determine the average normal stresses  $\bar{\sigma}_{xx}$  and  $\bar{\sigma}_{yy}$  and the average axial strains  $\bar{\epsilon}_{xx}$  and  $\bar{\epsilon}_{yy}$ . The homogenised Poisson's ratio and elastic modulus are calculated from the following expressions:

$$\nu = \frac{\bar{\sigma}_{yy} \bar{\epsilon}_{xx} - \bar{\sigma}_{xx} \bar{\epsilon}_{yy}}{\bar{\sigma}_{xx} \bar{\epsilon}_{xx} - \bar{\sigma}_{yy} \bar{\epsilon}_{yy}}, \quad (1)$$

$$E = \frac{\bar{\sigma}_{xx}^2 - \bar{\sigma}_{yy}^2}{\bar{\sigma}_{xx} \bar{\epsilon}_{xx} - \bar{\sigma}_{yy} \bar{\epsilon}_{yy}}. \quad (2)$$

We obtain  $\nu = -0.00156 \approx 0$  and  $E = 81.3$  GPa. The Young's modulus of the porous structure is obviously smaller than that of the matrix for the presence of the holes.

If the orientation angle  $\theta$  is modified while keeping the length of the holes  $a$  fixed, the behavior of the perforated sheet can be affected significantly. For instance, if  $\theta = 0^\circ$  the porous structure exhibits a positive Poisson's ratio (see Fig. 2b). On the other hand, if the value of  $\theta$  is not changed whereas  $a$  is increased, the Poisson's ratio of the medium becomes negative (see Fig. 2c). The deformations of the porous structure under stretching and compression in the three different cases investigated in Fig. 2 are better illustrated in the videos accompanying this paper (see Video 1–Video 3 in the Supplementary Material).

### 2.2. Periodic structure

Now we assume that the perforated sheet is of infinite extent, so that we can study a single elementary cell with periodic conditions at the boundaries. We determine the homogenised properties of the cell by applying a macroscopic uniaxial strain  $\bar{\epsilon}_{xx} = 10^{-4}$ , which generates local stresses below the yield limit. Accordingly, the periodic conditions are given as follows (refer to Fig. 1c):

$$\begin{aligned} u|_{BC} &= u|_{AD} + \bar{\epsilon}_{xx} \sqrt{3}l, & v|_{BC} &= v|_{AD}, & u|_{CD} &= u|_{AB}, \\ v|_{CD} &= v|_{AB}. \end{aligned} \quad (3)$$

In the formulae above,  $u$  and  $v$  are the horizontal and vertical components of the displacement field, respectively. In order to compute the average values of the normal stresses  $\bar{\sigma}_{xx}$  and  $\bar{\sigma}_{yy}$ , we build a finite element model in *Comsol Multiphysics*<sup>®</sup>, which has a very fine mesh with around 25,000 triangular elements. Then, we calculate the homogenised Poisson's ratio  $\nu$  and Young's modulus  $E$  from Eqs. (1) and (2), respectively. We find  $\nu = 0.00132 \approx 0$  and  $E = 83.1$  GPa, which are very close to the values obtained for the finite structure (the small discrepancies are due to the boundary layer effects). The same results are derived by applying a macroscopic uniaxial strain  $\bar{\epsilon}_{yy} = 10^{-4}$  or a macroscopic shear strain  $\bar{\epsilon}_{xy} = 10^{-4}$ , since the medium is isotropic in the plane.

The periodic elementary cell is used to perform a parametric study on the geometrical and constitutive properties of the structure. For instance, it is interesting to investigate the effects of the orientation angle  $\theta$  on the behavior of the medium. To this aim, we fix the elastic constants of the matrix ( $E_m = 300$  GPa,  $\nu_m = 0.36$ ) and we determine – for different values of the orientation angle  $\theta$  – the ratio  $a/l$  which yields a null Poisson's ratio. The outcomes are shown in Fig. 3a by the circles, while the squares represent the limit values

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