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On the optical bistability of ferroelectrics via two-wave mixing: Beyond the slowly varying envelope approximation

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ABSTRACT

In this paper, we investigate the Optical Bistability (OB) resulting from a degenerate two-wave mixing process in a single layer of Kerr nonlinear ferroelectric material. The nonlinear process is mathematically described by a nonlinear coupled-wave (CW) equations propagating across the nonlinear medium. The system is solved numerically with and without employing the Slowly Varying Envelope Approximation (SVEA). The effects of layer thickness, operating frequency, and the intensity of the probe beam on the bistable response are investigated in both cases with and without SVEA. We have found that there is a significant difference between both solutions (with and without SVEA) in the analysis of the CW theory of two-wave mixing for OB. The SVEA solution fails to predict the bistable response for various combinations of the input parameters. Furthermore, the solution with SVEA overlooks the detection of several bistable response periods, reduces the switching contrast of the detected OB, and shows only higher threshold values of bistable response. A comparison between our theoretical results and some available experimental data is also shown.

1. Introduction

Optical bistability (OB) is a third-order nonlinear optical effect where a system exhibits more than one output for each individual input. OB is believed to have promising applications as basic elements for future optical computers and optical switches. To generate a bistable system, a third-order nonlinear material (absorptive or dispersive) and a feedback loop is required. In our previous studies, we have demonstrated the existence of OB and multistability in the case of one THz laser beam interacting with a third-order Kerr nonlinear dielectric [1] and ferroelectric (FE) [2] materials. In case of a single layer with one beam, the incident wave modifies the material properties at the microscopic level, which provides an intrinsic feedback mechanism. i. e. Every molecule responds nonuniquely to the driving field [3,4]. In a Fabry-Perot resonator the feedback is supplied by the external cavity enclosing the medium, i.e. due to the constructive interference between several modes of oscillation inside the cavity [5–7]. On the other side, Previous studies have utilized a two-wave mixing (TWM) process to generate a feedback mechanism via the coupling of these waves inside the nonlinear medium. The wave mixing process can be distinguished into two types; degenerate and nondegenerate. In the degenerate case, two or more coherent beams interact with each other while the nondegenerate case involves beams with different frequencies. In theory, when two beams mix, the energy is transferred from one beam to the other and as a result, an interference pattern is formed in the material. In the degenerate case, a stationary interference pattern is formed while the nondegenerate case resulting in a shifting interference pattern [8,9].

The Slowly Varying Envelope Approximation (SVEA) is known to be a priori for investigating the optical propagation problems in guiding structures and have been utilized in most of the paper concerning the wave mixing problems [10–12]. With SVEA, the variation of complex amplitude over time and space is assumed to be slow and the backward wave is negligible. In physical meaning, the optical period is considered to be much less than the time for electron-photon interaction. In coupled-wave (CW) theory, Crosignani et al. [13] have shown that without employing the SVEA, the exact solution in terms of the second-order CW equations are identical to the exact solution of the first-order CW equations derived directly from Maxwell's equations. Furthermore, the authors in Ref. [13] have found that the SVEA assumption is not necessary in the analysis of the CW theory, that is to say, there is no need to invoke any condition of a slow variation with the propagation coordinates in order to ignore the second-order terms. In Ref. [14], the limit of validity of the SVEA is generalized to the condition $l_p \gg \lambda(1 - v/c)$ where v is the group velocity of the

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radiating system, λ is the operating wavelength, and l_p is the radiation pulse length. This condition reduces to the usual SVEA in the limit $v/c \ll 1$. Perhaps that is why the validity of the SVEA differs from one case to the other for optical propagation problems in guiding structures. In CW theory, we believe that in general, the second derivative terms definitely contribute some information about the behavior of the fields [15]. The importance and criticality of this information depend on the nature of the problem and the combination of the input set of parameters. For example, when the CW formalism is applied to the theory of volume diffraction gratings, the CW equations yield significantly different results for certain range of parameters [16,17]. Another clear example is the influence of SVEA approximation on the dynamics of the high-gain lasers as demonstrated in [18–22].

To our knowledge, all previous mathematical models of the OB via a TWM process are all obtained under the SVEA approximation. Hence, it is interesting to examine the validity of the SVEA approximation for the problem of OB in FE using a TWM process. To do so, we have directly solved the second-order CW equations numerically without SVEA and compared their results with the solution of the first order equations. We have focused on the third-order nonlinear Kerr effect where the refractive index of the material changes in response to the optical field. In particular, we assume two degenerate beams interacting with a single FE layer as shown in Fig. 1. Using Maxwell's wave equation and the nonlinear wave mixing process, a CW system describes the TWM process in the medium is derived. The current mathematical approach is unique in the sense that it describes the third-order nonlinearity in the CW system as a direct function of the Third-order nonlinear susceptibility $\chi^{(3)}$ of the material. Furthermore, $\chi^{(3)}$ is directly estimated from the available experimental and theoretical data of $BaTiO_3$. We shall show that when the SVEA is used, the CW equations fail to predict several bistable regions as well as the multistable response of the system. We believe that the results obtained here are more suitable to describe the experimental OB observed in FE materials using the TWM process. In general, the use of the SVEA leads to a considerable reduction of the computation time, but introduces significant error in the detection of the OB response of the system. For demonstration, a comparison between our theoretical results and some available experimental data in the literature will be shown.

The TWM theory has been mostly applied to test several other phenomena, material properties, and applications concerning the interaction of light with matter. To the best of our knowledge, the only theoretical study to realize the OB due to a TWM process using CW theory was done by Zozulya et al. [10]. Shortly after that, the experimental observation of OB in photorefractive Strontium Barium Niobate ($Sr_xBa_{1-x}Nb_2O_6$, SBN) via a TWM process with feedback was reported by Baraban et al. [11]. In addition, several experiments have observed the OB via a TWM process on barium titanate ($BaTiO_3$) as well [12,23,24]. The common feature of those experimental papers involving the TWM process in FE [For e.g. 11,12] is the clear mismatch between the experimental data and the analytical solution obtained by Zozulya et al. [10] under SVEA. Hence, the experimental and the theoretical results were shown in separate graphs. We believe that the cause is the employment of the SVEA in deriving the analytical solution by Zozulya et al. [10]. The semi-analytical approach presented here is based on the CW theory where the OB in FE can be obtained by an exact and an approximate method for the TWM process. The exact method is to directly solve the second-order differential CW equations and to find the OB output. The approximate solution is to use the SVEA to first change the second-order differential CW equations into the first-order ones and then to calculate the OB outputs. It is important to emphasize that the theoretical approach presented here is valid and can be applied to examine the OB for any photorefractive FE material not just $BaTiO_3$.

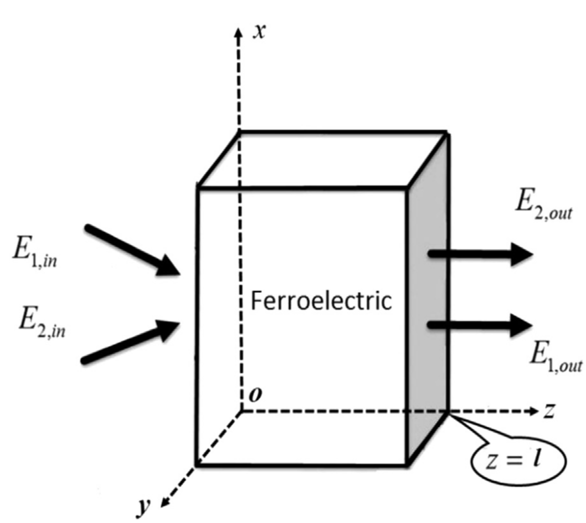

Fig. 1. Geometry of the two-wave mixing process where two laser beams interacts inside a Kerr ferroelectric medium.

2. Mathematical formulation

We assume a single FE layer illuminated by two optical waves. The electric field of both beams inside the medium form a standing wave pattern and can be expressed as time harmonic waves as;

$$\boldsymbol{E}_j(r, t) = \frac{1}{2}\left[E_j(r)\exp\left(-i\omega_j t\right) + c.c\right] \qquad j = 1, 2 \tag{1}$$

Each beam is characterized by its amplitude E and frequency ω where E_1 and ω_1 is the amplitude and frequency of the pump beam respectively, while E_2 and ω_2 is the probe-beam amplitude and frequency respectively and *c.c.* is the complex conjugate. For simplicity, we assume the case of normal incidence, and both medium 1 and 3 are free space. To investigate the coupling, we recall the following scalar wave equation for a time-dependent electric field propagates in a nonlinear medium:

$$\nabla^2\boldsymbol{E}(r, t) = \epsilon_{r\infty}\mu_0\epsilon_0\frac{\partial^2\boldsymbol{E}(r, t)}{\partial t^2} + \mu_0\frac{\partial^2\boldsymbol{P}^{NL}(r, t)}{\partial t^2} \tag{2}$$

The above form of the wave equation is obtained naturally from the general form of Maxwell's equation under the absence of current and charge densities. In Eq. (2), ε_∞ is the high-frequency limit of the linear dielectric function, c and t are the velocity of light in free space and time respectively. ***P****(r,t)* is the induced nonlinear polarization inside the medium. Conventionally, the nonlinear polarization may be expressed as Taylor series expansion in $\boldsymbol{E}$ where the third-order term represents the third-order nonlinearity. Here, for the two-wave mixing process, the third-order term can be written as;

$$\boldsymbol{P} = \epsilon_0\chi^{(3)}\left\{\frac{1}{2}\left[E_1(r)\exp\left(-i\omega_1 t\right) + c.c\right] + \frac{1}{2}\left[E_2(r)\exp\left(-i\omega_2 t\right) + c.c\right]\right\}^3 \tag{3}$$

The coefficient $\chi^{(3)}$ is the third-order nonlinear susceptibility of the medium while ϵ_0 is the vacuum permittivity. In fact, a few nonlinear optical processes are obtained from Eq. (3) Such as third-harmonic generation, sum and difference frequency generation. These terms are ignored and only the intensity-dependent terms are considered. Under the assumption of phase matching with Self Phase Modulation (SPM) and Cross Phase Modulation