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Invited Paper

Broadband and ultra-broadband modular half-wave plates

Emiliya Dimova [a], Wei Huang [b], George Popkirov [c], Andon Rangelov [b,d,*], Elica Kyoseva [b]

[a] Institute of Solid State Physics, Bulgarian Academy of Sciences, 72 Tsarigradsko Chaussée Blvd., 1784 Sofia, Bulgaria
[b] Engineering Product Development, Singapore University of Technology and Design, 8 Somapah Road, 487372 Singapore, Singapore
[c] Central Laboratory for Solar Energy and New Energy Sources, Bulgarian Academy of Sciences, 72 Tsarigradsko Chaussé Blvd., 1784 Sofia, Bulgaria
[d] Department of Physics, Sofia University, James Bourchier 5 blvd., 1164 Sofia, Bulgaria

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ABSTRACT

We experimentally demonstrate broadband and ultra-broadband spectral bandwidth modular half-wave plates. Both modular devices comprise an array of rotated single half-wave plates (HWPs), whereby for broadband and ultra-broadband performance we use standard and commercial achromatic HWPs, respectively. The bandwidth of the modular HWPs depends on the number N of individual HWPs used and in this paper we experimentally investigate this for $N=\{3,5,7,9\}$. The elements in the arrays are rotated at specific angles with respect to their fast-polarization axes, independent of the nature of the birefringent material. We find the rotation angles using an analogy to the technique of composite pulses, which is widely used for control in nuclear magnetic resonance.

1. Introduction

A retarder or a waveplate is an optical device that changes the polarization of a light beam passing through it [1–5]. The half-wave plate, which is one of the two most commonly used waveplates, rotates the polarization vector of an incident light at 2θ, where θ denotes the angle between the incident light polarization direction and the fast axis of the waveplate. Thus, a half-wave plate can be used to change the direction of linearly polarized light from horizontal to vertical, or vice versa, or to invert the handedness of light with circular polarization [1–5]. A big disadvantage of traditional waveplates is that they exhibit strong wavelength dispersion, mainly due to their fixed thickness. Therefore, half-wave plates perform as desired only for a very narrow range of wavelengths around the wavelength they have been designed for.

The effective wavelength dispersion of waveplates can be reduced by introducing an additional medium with reciprocal dispersion, a technique widely used for the production of commercial achromatic waveplates [6–9]. Another approach to reduce the dispersion is to use a stack of waveplates whose optical axes are rotated at given angles creating a sequence of phase shifts and thereby near constant retardation over a large bandwidth or for a larger set of discrete wavelengths [10–14]. Most recently, the analogy between stacks of waveplates and composite pulses in nuclear magnetic resonance [15] was used by several authors [16–21] to propose and experimentally demonstrate the applicability of a calculation method to derive broadband polarization waveplates.

In this paper we reveal the applicability of the composite pulses method [16–21] to design modular broadband and ultra-broadband half-wave plates irrespective of the optical medium used. We experimentally demonstrate that the modular half-wave plates operate over a wide range of wavelengths and that their performance depends strongly on the number of individual HWPs used. For our measurements we stack 3, 5, 7 and 9 standard and achromatic waveplates to demonstrate broadband and ultra-broadband performance, respectively.

2. Theory

Below we review in a concise manner the basic theory of composite pulses as applied to polarization waveplates. In the Jones calculus a single birefringent retarder rotated at an angle θ with reference to the slow and the fast axes of the plate is described by the Jones matrix,

$$\mathfrak{J}_{\theta}(\varphi)=\mathfrak{R}(-\theta)\begin{bmatrix} e^{i\varphi/2} & 0 \\ 0 & e^{-i\varphi/2}\end{bmatrix}\mathfrak{R}(\theta), \tag{1}$$

* Corresponding author at: Engineering Product Development, Singapore University of Technology and Design, 8 Somapah Road, 487372 Singapore, Singapore.
E-mail address: rangelov@phys.uni-sofia.bg (A. Rangelov).

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where

$$\Re(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \quad (2)$$

and the phase retardance is $\varphi = 2\pi L(n_f - n_s)/\lambda$, with λ being the vacuum wavelength, n_f and n_s the refractive indices along the fast and slow axes of the bierefringent medium and L the thickness of the retarder. When the phase retardance is $\varphi = \pi$, then the retarder is a half-wave plate.

In the left-right circular polarization (LR) basis the Jones matrix of the retarder is transformed according to $\mathbf{J}_\theta(\varphi) = \mathbf{W}^{-1}\mathfrak{J}_\theta(\varphi)\mathbf{W}$, where $\mathbf{W}$ links the horizontal–vertical polarization (HV) basis and LR polarization basis,

$$\mathbf{W} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix}. \quad (3)$$

Explicitly, the Jones matrix for a retarder with a phase retardance φ and rotated by an angle θ is given as (in the LR basis),

$$\mathbf{J}_\theta(\varphi) = \begin{bmatrix} \cos(\varphi/2) & i\sin(\varphi/2)e^{2i\theta} \\ i\sin(\varphi/2)e^{-2i\theta} & \cos(\varphi/2) \end{bmatrix}. \quad (4)$$

Our goal is to design a half-wave plate that is robust to variations in the phase retardance φ around values $\varphi = m\pi$, $(m \in \mathbb{N})$. Such half-wave plates are insensitive to imperfect rotary power φ/L and deviations in the plate thickness L, and furthermore, operate over a wide range of wavelengths λ. To achieve our goal, we follow the method of composite pulses [17,19–21], which is widely adopted for robust control in quantum optics [22,23]. In detail, we replace the single half-wave plate with an array of an odd number $N = (2n + 1)$ half-wave plates, thereby creating a half-wave plate with modular design. Each individual waveplate (module) has a phase retardance $\varphi = \pi$ and is rotated at an angle θ_k given the "anagram" condition $\theta_k = \theta_{N+1-k}$, $(k = 1, 2, \ldots, n)$. The Jones matrix of the above described arrangement of waveplates in the LR basis is given by (read from right to left):

$$\mathbf{J}^{(N)} = \mathbf{J}_{\theta_N}(\varphi)\mathbf{J}_{\theta_{N-1}}(\varphi)\cdots\mathbf{J}_{\theta_1}(\varphi). \quad (5)$$

We aim to implement an ideal half-wave plate propagator with Jones matrix $\mathbf{J}_0$ in the LR basis (up to a global phase factor),

$$\mathbf{J}_0 = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}, \quad (6)$$

through the modular half-wave plate propagator with Jones matrix $\mathbf{J}^{(N)}$ from Eq. (5). That is, we set $\mathbf{J}^{(N)} \equiv \mathbf{J}_0$ at $\varphi = \pi$, which allows us to use the n independent angles θ_k as control parameters. We then nullify as many lowest order derivatives of $\mathbf{J}^{(N)}$ vs. the optical retardance φ at $\varphi = \pi$ as possible. We thus obtain a system of nonlinear algebraic equations for the rotation angles θ_k:

$$\left[\partial_\varphi^k \mathbf{J}_{11}^{(N)}\right]_{\varphi=\pi} = 0, \quad \left[\partial_\varphi^k \mathbf{J}_{12}^{(N)}\right]_{\varphi=\pi} = 0, \quad (7)$$

where $(k = 1, 2, \ldots, n)$. The anagram symmetry assumption for the angles θ_k $(\theta_k = \theta_{N+1-k})$, ensures that all even-order derivatives of $\mathbf{J}_{11}^{(N)}$ and all odd-order derivatives of $\mathbf{J}_{12}^{(N)}$ vanish. Hence, the n θ_k angles allow us to nullify the first n derivatives of the matrix $\mathbf{J}^{(N)}$ (5).

Solutions to Eqs. (7) give the recipe to construct *arbitrary broadband* modular half-wave plates. A larger number N of individual half-wave plates provides a higher order of robustness against variations in the phase shift φ and thus, the light wavelength λ. We list several examples of modular broadband half-wave plates designs in Table 1.

We define the fidelity of the Jones matrix of the modular half-wave plate according to

$$F = \frac{1}{2}\mathrm{Tr}(J_0^{-1}J^{(N)}). \quad (8)$$

We calculate the spectral performance called further "fidelity" as a function of the wavelength and give the results for broadband and

Table 1
Rotation angles θ_k (in degrees) for modular broadband half-wave plates with different numbers N of constituent half-wave plates.

N	Rotation angles $(\theta_1; \theta_2; \ldots; \theta_{N-1}; \theta_N)$
3	(60; 120; 60)
5	(51.0; 79.7; 147.3; 79.7; 51.0)
7	(68.0; 16.6; 98.4; 119.8; 98.4; 16.6; 68.0)
9	(99.4; 25.1; 64.7; 141.0; 93.8; 141.0; 64.7; 25.1; 99.4)

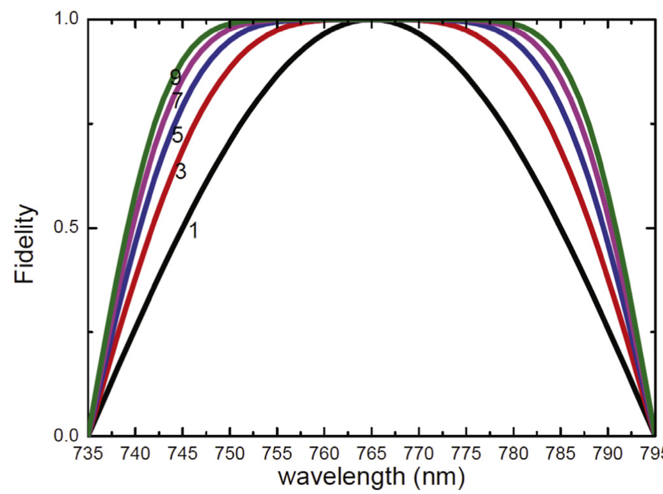

Fig. 1. Fidelity (8) of the Jones matrix of a linear modular half-wave plate arranged as a sequence of individual half-wave plates (WPMQ10M-780, Thorlabs) as a function of the wavelength λ. The number of individual half-wave plates was $N = \{3, 5, 7, 9\}$ and their respective optical axes were rotated at the angles given in Table 1. For easy reference we also include the fidelity of a single half-wave plate, labeled by 1.

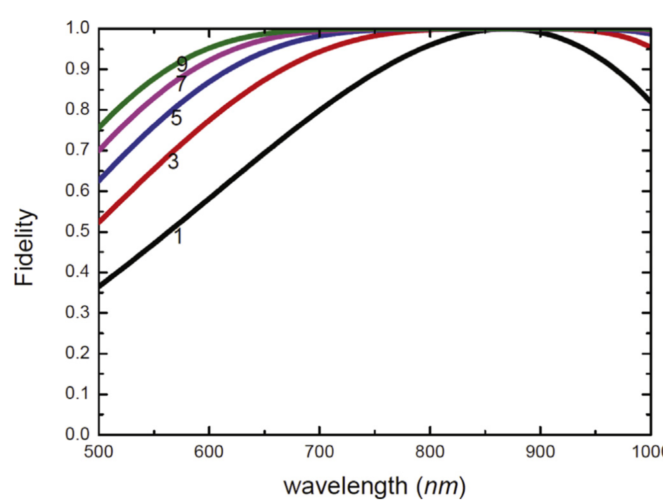

Fig. 2. Fidelity (8) of the Jones matrix of a modular half-wave plate consisting a sequence of commercial achromatic half-wave plates (WRM053-mica, 700–1100 nm) as a function of the wavelength. The number of achromatic half-wave plates used to assemble the modular half-wave plate was $N = \{3, 5, 7, 9\}$ and again the angles of rotation are given in Table 1. For easy reference we also include the fidelity of a single achromatic half-wave plate, labeled by 1.