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Frequency domain optical tomography using a Monte Carlo perturbation method

Toshihiro Yamamoto [a,*], Hiroki Sakamoto [b]

[a] Research Reactor Institute, Kyoto University, 2 Asashiro Nishi, Kumatori-cho, Sennan-gun, Osaka 590-0494, Japan
[b] Transnuclear Tokyo, Ltd., 1-18-16, Shinbashi, Minato-ku, Tokyo 105-0004, Japan

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ABSTRACT

A frequency domain Monte Carlo method is applied to near-infrared optical tomography, where an intensity-modulated light source with a given modulation frequency is used to reconstruct optical properties. The frequency domain reconstruction technique allows for better separation between the scattering and absorption properties of inclusions, even for ill-posed inverse problems, due to cross-talk between the scattering and absorption reconstructions. The frequency domain Monte Carlo calculation for light transport in an absorbing and scattering medium has thus far been analyzed mostly for the reconstruction of optical properties in simple layered tissues. This study applies a Monte Carlo calculation algorithm, which can handle complex-valued particle weights for solving a frequency domain transport equation, to optical tomography in two-dimensional heterogeneous tissues. The Jacobian matrix that is needed to reconstruct the optical properties is obtained by a first-order "differential operator" technique, which involves less variance than the conventional "correlated sampling" technique. The numerical examples in this paper indicate that the newly proposed Monte Carlo method provides reconstructed results for the scattering and absorption coefficients that compare favorably with the results obtained from conventional deterministic or Monte Carlo methods.

1. Introduction

Near-infrared optical tomography has been developed as a promising technique for the effective retrieval of spatially dependent optical parameters in a turbid medium, such as absorption and scattering coefficients. The technique reconstructs the optical parameters of an internal medium from the measured transmittance and reflectance at the boundary surface of a probed medium. The propagation of radiation in a medium is mainly calculated using the diffusion approximation theory or the exact radiation transport theory. It is well known that the diffusion approximation theory introduces inaccuracy in void-like regions, near the source, or at the boundary surface, but it is very cost-effective and easy to solve [1,2]. Representative methods for solving a radiative transfer equation include the discrete ordinates method and the Monte Carlo method. The research performed thus far has utilized the discrete ordinates method for the purpose of near-infrared optical tomography [1–14]. Some spatial discretization schemes, such as the finite volume or finite difference scheme, are commonly used and are associated with the discrete ordinates method. The finite element method has drawn increasing attention due to its flexibility in handling complex geometries.

Another promising calculation tool proposed here is the Monte Carlo method. The most notable advantage of the Monte Carlo method over deterministic methods is its lack of limitations when handling three-dimensional complex geometries. The Monte Carlo method also eliminates discretization in the dimensions of energy, time and angle. If a large number of particles are used, the Monte Carlo method is therefore free from the ray effects and false scattering that are major sources of inaccuracy in the discrete ordinates method. The disadvantage of the Monte Carlo method is that an estimate is always accompanied by statistical uncertainty and the wave properties of light is ignored. In general, the Monte Carlo method is thought to be overly time-consuming and expensive for reducing uncertainties below an acceptable level. However, with the advent of high performance CPUs, massive parallel computing technologies, and some of new Monte Carlo methods that are being developed, the drawbacks of the Monte Carlo method can be overcome, even for problems that were previously impossibly time-consuming.

Radiation transfer calculations for optical tomography are often performed in the frequency domain. In a frequency domain calculation, the radiation beam intensity is modulated in amplitude at a given frequency. By using a modulated radiance, phase

* Corresponding author.
E-mail address: toshihiro.yamamoto223@gmail.com (T. Yamamoto).

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information is available in addition to amplitude. When we seek to simultaneously reconstruct two optical coefficients, such as scattering and absorption coefficients, cross-talk between the two coefficients has been observed, and may lead to a wrong diagnosis [4]. To avoid cross-talk, additional information is needed. Frequency domain measurements would provide information regarding the phase of the radiation in addition to its intensity. Thus, frequency domain techniques allow for better separation of absorption and scattering effects [15].

The deterministic radiation transfer calculation method in the frequency domain is an established and common tool. To the best of our knowledge, there have not been many studies on the Monte Carlo method for optical tomography, especially in the frequency domain. A general survey is provided on the Monte Carlo modeling of radiation transfer in tissue optics in [16], which reviews the history and recent progress in the development of the Monte Carlo method in this area of research in detail. Some previous studies that performed frequency domain Monte Carlo calculations have been published in the literature [17–20]. In [17], the Monte Carlo method in the frequency domain is used to analyze the amplitude and phase delay of detected waves at a single frequency. This method is referred to as the “shortcut method” because a time domain calculation that includes many frequencies can be bypassed. The amplitude and phase at a single frequency can be measured or calculated using a temporal Fourier transform of the time series data for the detected signals. The shortcut method performs a Monte Carlo calculation only for a single modulation frequency. In [18,19] radiative signals in the frequency domain are obtained via the Monte Carlo method following the techniques presented in [17], and provide solutions to inverse photon migration problems in heterogeneous turbid media. In [20], a spatial Fourier transform is performed on the radiative transfer equation to analyze a layered tissue system. The complex radiation transport equation is solved via the Monte Carlo method using complex-valued particle weights.

There is another advantage when treating a radiative transfer equation in the frequency domain: the time-dependent radiative transfer equation can easily be solved in the frequency domain because no time-derivative term exists in the frequency domain equation [21,22]. At first, the time-dependent radiative transfer equation is transformed into the frequency domain equation by applying a Fourier transform. The complex-valued frequency domain equation is solved for each frequency that is contained in the incident light source. Time-dependent results in the time domain can be obtained by applying an inverse Fourier transform to the results from the frequency domain.

The authors recently applied this deterministic technique to solve a time-dependent neutron transport equation in a subcritical nuclear reactor [23] using the Monte Carlo method. The neutron transient behavior is induced by the time variation of the neutron source intensity. The complex-valued neutron transport equation in the frequency domain is solved for each frequency contained in the neutron source time variation to obtain the complex-valued neutron flux. By applying an inverse Fourier transform to the complex-valued neutron flux, the time variation of the neutron flux is obtained.

Previous studies performed by Hayakawa et al. [18], Seo et al. [24], Zhao et al., [25,26] use the Monte Carlo method in the frequency domain for solving inverse problems to determine optical properties. A similar study was performed by Sharma et al. [27], not for a modulated source but for steady-state diffuse reflectance in two-layered phantoms. These studies address homogeneous tissue [25,26] or heterogeneous simple geometries [27] that are composed of a one-dimensional, two-layered tissue. Kumar and Vasu [28], and Yalavarthy et al. [29] performed studies to reconstruct the optical properties of a heterogeneous two-dimensional tissue model using the perturbation Monte Carlo technique. Their studies do not address the frequency domain problem and are limited to steady-state Monte Carlo modeling. The study in this paper aims to extend the Monte Carlo calculation algorithm developed for nuclear reactor kinetics calculations in the frequency domain to transient radiative transfer calculations, which can subsequently be available for optical tomography. This will allow reconstruction of the multi-dimensional distribution of optical properties, as performed in [4–13] using the deterministic methods.

2. Algorithm for the frequency domain Monte Carlo calculation method

In this section, we briefly review the method adopted in [23] that outlines how to solve the radiative transfer equation in the frequency domain using the Monte Carlo method. The time-dependent radiative transfer equation in a non-multiplying medium is written as [23]

$$\begin{aligned}\frac{1}{c}\frac{\partial}{\partial t}I(\mathbf{r},\mathbf{\Omega},t) &= -\mathbf{\Omega}\cdot\nabla I(\mathbf{r},\mathbf{\Omega},t)-(\mu_a(\mathbf{r})+\mu_s(\mathbf{r}))I(\mathbf{r},\mathbf{\Omega},t)\\ &+\frac{\mu_s(\mathbf{r})}{4\pi}\int_{4\pi}d\mathbf{\Omega}'f(\mathbf{r},\mathbf{\Omega}'\to\mathbf{\Omega})I(\mathbf{r},\mathbf{\Omega}',t)\\ &+S(\mathbf{r},\mathbf{\Omega},t),\end{aligned}\tag{1}$$

where $I(\mathbf{r},\mathbf{\Omega},t)$= the radiant power per unit solid angle per unit area at position $\mathbf{r}$ with direction $\mathbf{\Omega}$ and time t, c=the light speed within the medium, μ_a=the absorption coefficient, μ_s= the scattering coefficient, $f(\mathbf{r},\mathbf{\Omega}'\to\mathbf{\Omega})$= the angular distribution of the scattered radiation, and S=the radiation source term. This study chooses the Henyey-Greenstein function for the angular distribution of the scattered radiation as [10]

$$f(\mathbf{r},\mathbf{\Omega}'\to\mathbf{\Omega})=\frac{1-g^2}{(1+g^2-2g\cos\theta)^{3/2}},\tag{2}$$

where θ is the angle between $\mathbf{\Omega}'$ and $\mathbf{\Omega}$, and g is the anisotropy factor.

The external collimated beam that is often used in optical tomography penetrates the medium and is scattered within the medium in the course of its penetration. Due to strong discontinuities, radiative transport calculations are conventionally performed in two steps. In the first step, the collimated intensity that obeys the extinction law is obtained. In the second step, the scattered intensity induced by the scattering of the collimated intensity is obtained. However, for the Monte Carlo method, the discontinuity does not cause any difficulties within a small-sized medium treated in optical tomography. Thus, the collimated and scattered intensities are calculated simultaneously in this study.

The time domain equation, Eq. (1), is converted to a frequency domain equation via a Fourier transformation. We obtain the transport equation for the radiation intensity in the frequency domain [4–13]:

$$\begin{aligned}&\mathbf{\Omega}\cdot\nabla\tilde{I}(\mathbf{r},\mathbf{\Omega},\omega)+(\mu_a(\mathbf{r})+\mu_s(\mathbf{r}))\tilde{I}(\mathbf{r},\mathbf{\Omega},\omega)\\ &=\frac{\mu_s(\mathbf{r})}{4\pi}\int_{4\pi}d\mathbf{\Omega}'f(\mathbf{r},\mathbf{\Omega}'\to\mathbf{\Omega})\tilde{I}(\mathbf{r},\mathbf{\Omega}',\omega)\\ &-\frac{i\omega}{c}\tilde{I}(\mathbf{r},\mathbf{\Omega},\omega)+\tilde{S}(\mathbf{r},\mathbf{\Omega},\omega),\end{aligned}\tag{3}$$

where ω= angular frequency, $i=\sqrt{-1}$, and:

$$\tilde{I}(\mathbf{r},\mathbf{\Omega},\omega)\equiv\int_{-\infty}^{+\infty}I(\mathbf{r},\mathbf{\Omega},t)\,e^{-i\omega t}dt,\tag{4}$$