Optics Communications 355 (2015) 398–405

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Discussion

Aberrations in shift-invariant linear optical imaging systems using partially coherent fields

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ARTICLE INFO

Article history:
Received 4 March 2015
Received in revised form
29 June 2015
Accepted 30 June 2015
Available online 14 July 2015

Keywords:
Imaging
Aberrations
Partial coherence
Phase contrast

ABSTRACT

Here the role and influence of aberrations in optical imaging systems employing partially coherent complex scalar fields is studied. Imaging systems require aberrations to yield contrast in the output image. For linear shift-invariant optical systems, we develop an expression for the output cross-spectral density under the space-frequency formulation of statistically stationary partially coherent fields. We also develop expressions for the output cross-spectral density and associated spectral density for weak-phase, weak-phase–amplitude, and single-material objects in one transverse spatial dimension.

1. Introduction

When imaging transparent samples in an in-focus optical system such as a visible-light or x-ray microscope, the detected output image appears almost featureless if the system yields a reproduction of the input image that is incident upon the system [1]. This is what optics is commonly defined as a perfect or near perfect imaging system in which there are no transverse spatial variations within the incident spectral density distribution as it propagates to the output detection plane. Note that the term "spectral density" is here used in the sense of optical partial coherence. As perfect systems are unable to visualize the refraction effects (phase contrast) caused by transparent samples, the presence of aberrations is a necessary condition for non-negligible contrast in the output spectral density to be attained [2]. In this context, an aberrated imaging system may be defined as one whose output transverse spatial distribution of spectral density is not equal to the input transverse spatial distribution of spectral density, up to transverse and multiplicative scale factors together with the smearing effects of finite resolution. Almost all aberrated imaging systems exhibit phase contrast, i.e. have an output spatial distribution of spectral density which is influenced by the functional form of the input wavefronts (input phase distribution). Examples of aberrated imaging systems yielding phase contrast include Zernike phase contrast, propagation-based phase contrast, differential phase contrast, and inline holography [1,3–5].

Work relating to a partially coherent treatment specifically for propagation-based phase contrast imaging based on the Transport-of-Intensity equation has been reported [6–8]. In this paper we consider the generalized differential phase contrast associated with aberrated linear shift-invariant optical imaging systems employing statistically stationary partially coherent scalar radiation, for which the output spatial distribution of spectral density (i.e., the output image) can be modelled using the transfer function formalism. This extends previously reported work by Paganin and Gureyev [2] which restricted consideration to the generalized differential phase contrast of fully coherent scalar fields imaged using aberrated linear shift-invariant optical systems.

In Section 2 we obtain an equation that describes the action of shift-invariant linear systems using partially coherent fields, under the imaging assumption that the object under study is a pure thin phase object. A two-dimensional transverse Cartesian coordinate system is used in the derivation. In Section 3 expressions for the spectral density are derived, restricting consideration to only one transverse spatial variable for simplicity. Three different types of samples are considered: samples that satisfy (i) the weak-phase object approximation, (ii) the weak phase–amplitude approximation, and (iii) the single material weak phase–amplitude approximation. Section 4 studies in depth the features of the transfer function used in this formalism.

2. Shift-invariant, linear systems for partially coherent fields using two transverse spatial coordinates

In this section we derive an expression for partially coherent complex scalar fields imaged by an optical system that is shift-invariant and satisfies the property of linearity [10]. For such a

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http://dx.doi.org/10.1016/j.optcom.2015.06.075
0030-4018/

system, the output complex disturbance is related to the input complex disturbance by the transfer function formalism [10]. Since most image collecting is normally done using two dimensional Cartesian grids it is natural to utilize a two-dimensional Cartesian system (x,y) in all calculations.

Before incorporating the effects of partial coherence in our derivations, we recall first a description of shift-invariant linear systems for fully coherent complex scalar wave-fields which are governed by the transfer function formalism. For such optical systems the output field $\Psi_{out}(x, y)$ is related to the input field $\Psi_{in}(x, y)$ by a Fourier-space filtration that can be written in operator form as [2]

$$\Psi_{out}(x, y) = F^{-1}\overline{T}(k_x, k_y)F\left\{\Psi_{in}(x, y)\right\}. \tag{1}$$

Here, $\overline{T}(k_x, k_y)$ is the transfer function characterizing the optical system, (k_x, k_y) are Fourier conjugate coordinates dual to (x,y), F and F^{-1} respectively represent the forward and inverse Fourier transform operations, and all operators are taken to act from right to left. Thus, the above equation states that F is applied to the input field $\Psi_{in}(x, y)$, before multiplying by the transfer function $\overline{T}(k_x, k_y)$ and then applying the operator F^{-1}, so as to yield the output field $\Psi_{out}(x, y)$ (see Fig. 1).

In our derivation the forward and inverse Fourier transform operation conventions used are the following:

$$\widehat{G}(k_x, k_y) = \frac{1}{2\pi}\iint_{-\infty}^{\infty} dx\, dy\, G(x, y)e^{-i(k_x x + k_y y)}, \tag{2a}$$

$$G(x, y) = \frac{1}{2\pi}\iint_{-\infty}^{\infty} dk_x\, dk_y \widehat{G}(k_x, k_y)e^{i(k_x x + k_y y)}. \tag{2b}$$

Here, $\widehat{G}(k_x, k_y) \equiv F\left\{G(x, y)\right\}$.

To proceed further, we follow Paganin and Gureyev [2] and make the restricting assumption that the transfer function $\overline{T}(k_x, k_y)$ is sufficiently well behaved for its logarithm to admit a Taylor-series representation. Note that a necessary condition for this assumption to be valid is that the transfer function does not possess any zeros over the patch of Fourier space for which the modulus of $F\{\Psi_{in}(x, y)\}$ is non-negligible, a region which may be termed the "essential spectral support" of the input field.

While this key assumption will fail for imaging systems such as Schlieren optics which completely block certain spatial frequencies in the essential spectral support of the input disturbance, the assumption will hold for a variety of important imaging systems such as out-of-focus contrast [3], inline holography [5], interferometric phase contrast [11], differential phase contrast [12], and analyzer-based phase contrast of weakly scattering samples [4].

With the above in mind, our simplifying assumption allows us to express the transfer function in the classic form that is standard e.g. in transmission electron microscopy, namely [2,14,13]:

$$\overline{T}(k_x, k_y) = \exp\left(i\sum_{m,n=0}^{\infty}\widetilde{\alpha}_{mn}k_x^m k_y^n\right). \tag{3}$$

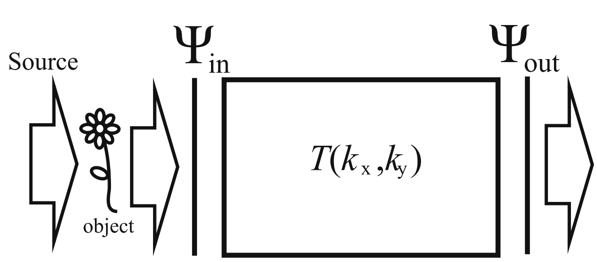

Fig. 1. Schematic illustration of the action of an aberrated shift-invariant linear optical system for imaging fully coherent complex scalar wave-fields, under the transfer function formalism. Input and output complex fields are related by the transfer function formalism according to Eq. (1).

Under this representation we denote the set of complex numbers $\{\widetilde{\alpha}_{mn}\}$ as the "aberration coefficients" where m and n are non-negative integers and label the order of the aberration. The real part of each such coefficient is termed as a coherent aberration, with the corresponding imaginary part being termed as an incoherent aberration. See Paganin and Gureyev [2] for a direct link between these complex aberration coefficients, and the Siedel aberrations [15] (e.g., piston, defocus, astigmatism, spherical aberration, and chromatic aberration) of classical aberration theory.

Expanding the complex exponential in Eq. (3) as a Taylor-series, we obtain

$$\overline{T}(k_x, k_y) = 1 + i\sum_{m,n=0}^{\infty}\alpha_{mn}k_x^m k_y^n. \tag{4}$$

The above expression serves to define the set of coefficients $\{\alpha_{mn}\}$. The set of coefficients $\{\alpha_{mn}\}$ is defined in terms of the set of aberration coefficients $\{\widetilde{\alpha}_{mn}\}$. We note that like Eq. (3), Eq. (4) disallows the presence of any zeros in the transfer function $\overline{T}(k_x, k_y)$. This form is particularly useful for studying the effect of transfer functions which differ only slightly from unity, namely for weakly aberrated shift-invariant imaging systems. We shall pick up on this point later in the paper.

It is useful to write the operator form of Eq. (1) in terms of the following integral:

$$\Psi_{out}\left(x, y\right) = \frac{1}{2\pi}\iint_{-\infty}^{\infty} dk_x\, dk_y\overline{T}(k_x, k_y)e^{i(k_x x + k_y y)} \times \widehat{\Psi}_{in}(k_x, k_y), \tag{5}$$

where $\widehat{\Psi}_{in}(k_x, k_y)$ denotes the Fourier transform of $\Psi_{in}(x, y)$ with respect to x and y. The above integral-form expression describes the output wave-field for an optical system that is linear and shift-invariant for incoming wave-fields that are fully coherent.

We now turn to the extension of this theory of fully coherent fields to partially coherent fields. This corresponds to the generalization shown in Fig. 2. Here, W_{in} is the cross-spectral density incident upon a linear shift-invariant aberrated optical system, yielding the corresponding output cross-spectral density W_{out}.

Under the space-frequency description of partial coherence developed by Wolf [15,16], the output cross-spectral density at a specified angular frequency ω may be constructed using an ensemble of strictly monochromatic fields all of the same angular frequency, via:

$$W_{out}\left(x_1, y_1, x_2, y_2\right) = \left\langle\Psi_{out}^{*}(x_1, y_1)\Psi_{out}(x_2, y_2)\right\rangle_{\omega}. \tag{6}$$

Here, angular brackets denote the ensemble average. Note that one may also consider expressing W_{out} in terms of its coherent mode expansion but incorporating other such correlating descriptions into our framework is beyond the scope of this paper [15].

Putting this equation to one side for the moment, note that we

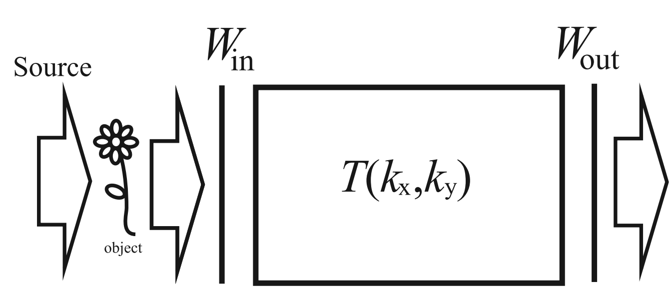

Fig. 2. Schematic illustration of the action of an aberrated shift-invariant linear optical imaging system, for statistically stationary partially coherent complex scalar fields, under the transfer function formalism. Input and output cross-spectral densities, W_{in} and W_{out} respectively, are related by the generalized transfer function formalism according to Eq. (8).