Optics Communications 345 (2015) 99–104

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

The spectral maps of dynamic speckle patterns: A new perspective to look at the motion-induced spatiotemporal cross-correlation

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ARTICLE INFO

Article history:
Received 13 November 2014
Received in revised form
23 January 2015
Accepted 29 January 2015
Available online 4 February 2015

Keywords:
Dynamics light scattering
Dynamics speckles
Spatiotemporal correlation
Spectral analysis

ABSTRACT

We introduce the spectral map method as a means to visualize the spatiotemporal correlation in dynamic speckles. Dynamic speckle patterns of moving diffuse objects are Fourier transformed to construct spectral maps in spatial domain (r-space) and spatial-frequency domain (k-space). The periodic patterns in r-space spectral maps demonstrate the spatiotemporal cross-correlation induced by the directional motions. To resolve the spectral bands in k-space spectral maps, and to observe the concomitant appearance of the periodic pattern in r-space spectral maps, the frequency must exceed the minimum values given by the speckle size and the speed. A bigger speckle size helps to reveal fast dynamics, and to ease the requirement on the minimum frequency. The transition from ordered translational speckles to chaotic boiling speckles at different speeds and speckle sizes is elucidated in connection with the change observed in spectral maps.

In dynamic light scatterings, the dynamics of the scatterers is encoded in the temporal fluctuation of the scattered light, and can be inferred by analyzing the autocorrelation function of time-varying light intensity or its counterpart in frequency domain: the power spectrum. In the spatial domain, the scattering of coherent waves by diffuse objects forms random speckle pattern, which also varies with time when the objects are in motion. By using multi-channel detectors, the time-series of fluctuating intensity can be recorded at each pixel of the detector, and the subsequent ensemble-averaging-based analysis allows the autocorrelation function or the power spectrum to be determined. As the technology of multichannel detectors improves, they have been increasingly employed to conduct dynamic light scattering experiments in fields such as laser microrheology [1–5]. Although the ensemble averaging offers a straightforward and reliable means to reduce the dimensionality of the data, it also throws out a great deal of information embedded in r-space and k-space of the dynamic speckle patterns.

More specifically, in cases of translational motions, oscillations and directional flows etc., the dynamic speckle patterns often appear to vary in a random and uncorrelated fashion despite of the fact that the scatterers move in an organized and ordered way. This phenomenon cannot be explained solely based on the temporal autocorrelation function, but is explained by the space-time cross-correlation of intensity fluctuations [6,7]. The spatiotemporal cross-correlation function is traditionally defined as the second-order intensity correlation in space and time domain [8,9]. Consequently, the ensemble averaging has to be performed to compute the correlation from the experimental data. Recent researches have extended the spatiotemporal cross-correlation approach to investigate the dynamic speckles produced with partially coherent illumination [10], and to study the dynamics of soft systems using multi-channel detectors [11,12]. In this work, the spatiotemporal correlation is analyzed in the temporal frequency and spatial frequency domain, as oppose to the traditional space-time domain approach. To this end, the spectrum of the time-series recorded at each pixel is obtained via the Fourier transform. Instead of ensemble averaging the individual spectrums over all pixels, the spatial distribution of a frequency component of the spectrum, i.e. the spectral map, is analyzed. Watson et al. have analyzed the motion of regular images in frequency domains [13], and k-space correlation spectroscopy has been applied in the analysis of fluorescent images [14]. Owning to the lack of a clearly defined image pattern and the randomness of the intensity fluctuation, the analysis of objective dynamic speckles presents a far more challenging problem. We demonstrate that the speckle size plays a crucial role in overcoming these inherent difficulties associated with the speckle phenomena.

The far-field free-space geometry is shown in Fig. 1a, where a frost glass plate serves as the diffuse object and is moved by a translational stage along the horizontal direction (i.e. x-direction).

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http://dx.doi.org/10.1016/j.optcom.2015.01.072
0030-4018/

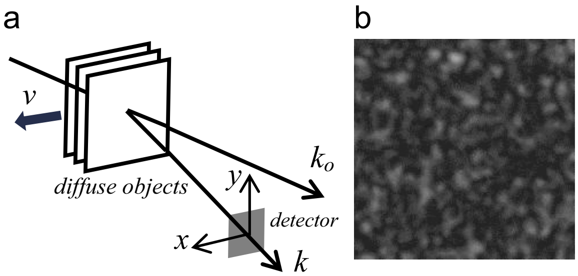

Fig. 1. (a) The schematic diagram of the scattering geometry, where k_o and k are the incident and the scattered wave-vectors. (b) A typical static speckle pattern.

The 632.8 nm emission of a He–Ne laser is scattered into the forward direction. The TEM laser beam is expanded to a diameter of 9 mm to illuminate sufficient number of scatterers on the plate. A CMOS detector with a pixel size of 5.3 µm (Thorlabs) is placed at a distance of 500 mm from the diffuser, and at 5° scattering angle with respect to the forward direction. An iris is placed at 100 mm in front of the detector, acting as the limiting aperture stop of the system. The dynamic speckle patterns are recorded for a region of interest (ROI) consisting of 200×200 pixels, at a frame rate of 257 Hz. Fig. 1b shows the static speckle pattern where the average speckle size is about 15 times of the pixel size. Because the angle subtended by the area of ROI to the scattering center is smaller than 0.2°, all pixels within the ROI are located at essentially the same scattering angle. Consequently, the time-averaged spectrums obtained at different pixels show no appreciable difference from each other.

For translational speckles, and the recorded intensity can be written as: $I(x - vt, y)$, where v is the speed of the translational motion. Performing the Fourier transform of the time-series recorded at each pixel, and applying the shift theorem, we get the spectral map in spatial-domain as

$$S(x,\ y,\ \omega) = \mathcal{F}_t[I(x - vt,\ y)] = I(x,\ y) \exp(-i\omega x/v) \tag{1}$$

where $S(x, y, \omega)$ is the r-space spectral map, and $\mathcal{F}_t$ denotes the Fourier transform with respect to time. Eq. (1) shows that $S(x, y, \omega)$ of a pure translational speckle pattern is simply its corresponding static speckle pattern $I(x, y)$ modulated by a harmonic function in the direction of motion. The frequency of the spatial modulation is proportional to temporal frequency ω and inversely proportional to the speed v.

In Fig. 2a and b, the real parts of $S(x, y, \omega)$ are shown at two different values of ω for a moving speed of 0.5 mm/s. The periodic features in the maps are apparent, and their spatial frequency increases with increasing temporal frequency. The profile plot along the horizontal line in Fig. 2a clearly shows the sinusoidal modulation in x-direction (e). The spectral maps for a moving speed of 1 mm/s are shown in Fig. 2c and d. Again, the periodic modulation can be seen to align in the x-direction. At the same ω, the spatial frequency in Fig. 2d is about half that of the corresponding spatial frequency in b, which is in agreement to the prediction of Eq. (1). The linear relation between the spatial and temporal frequency is made clear in Fig. 2f, where the slopes of fitted lines for 0.5 mm/s and 1 mm/s are different by a factor of 2 as expected. These observations unequivocally demonstrate that the periodicities in spatial and time-domains are correlated, and it is the translational motion that gives rise to the spatiotemporal cross-correlation.

The ensemble-averaged power spectrums are shown in Fig. 3a, which display typical spectral features observed in homodyne spectrums [15]. As can been seen from the scale of the color bars in Fig. 2, the amplitude of the spectrum decreases with increasing ω, which is expected based on the fact that the power spectrums fall off towards higher ω. Consequently, the signal-to-noise ratio (S/N) drops with increasing ω, which is reflected in the increasing grainy appearance in spectral maps (Fig. 3b). The main sources of noise include the sensor dark current and the shot noise. Being uncorrelated white noises, they vary at the pixel level, giving rise to the fine grainy appearance. Because the presence of the noise reduces the visibility of the periodic pattern, the level of the noise floor in the power spectrum imposes an upper limit on the frequency used to construct the spectral maps.

To investigate multiple scattering situations involving both static and dynamic scatterings, two additional frost glass plates are placed at the two sides of the moving plate to serve as static diffusers. When the middle plate undergoes the translational motion, a mix of translation and boiling speckle behavior is observed, and the boiling speckle becomes predominant with increasing moving speed. Due to the additional static scatterings, the actual scattering angle is no longer set by the position of the detector; but covers a much broader angular range which causes the speckles to fluctuate at much faster rates [15], as is evident in the much broader power spectrums in Fig. 3b. The average speckle size is reduced as a consequence of the broader angular range involved in the scattering [16]. As will be seen, both the faster dynamics and smaller speckle size contribute to the appearance of boiling speckles. The motion of the diffuser does not induce any appreciable change in the average speckle size. In addition, the contrast of the speckle patterns is not affected by the additional diffusers and the motion.

Despite of these differences, the spectral maps are still characterized by similar periodic modulation, provided that the temporal frequency at which the map is constructed exceeds certain minimum value. In Fig. 4, the real part of $S(x, y, \omega)$ is shown for a moving speed of 0.1 mm/s. The spectral map in Fig. 4a is constructed at $\omega = 56.5$ rad/s, and shows no obvious periodicity in any direction. At $\omega = 94.2$ rad/s (Fig. 4c) the periodic modulation begins to emerge, and at $\omega = 226.2$ rad/s (e) the pattern can be seen to align along the x-direction. Like in the previous case, the spatial frequency of the periodic feature increases with increasing temporal frequency ω, and the linearly relation between the two is verified again (Fig. 5a).To facilitate the identification and analysis of the periodic spatial feature, 2D spatial Fourier transform is performed to convert the r-space spectrum map to its counterpart in k-space. Applying the result of Eq. (1), we have

$$\begin{aligned}\tilde{S}\left(f_x, f_y, \omega\right) &= \mathcal{F}_{xy}[I(x, y) \cos(x\omega/v)] \\ &= \tilde{I}(f_x, f_y)\left\{\delta\left(f_x - \omega/2\pi v\right) + \delta(f_x + \omega/2\pi v)\right\}\end{aligned} \tag{2}$$

where $\tilde{S}(f_x, f_y, \omega)$ is the spectral map in spatial-frequency domain, and $\mathcal{F}_{xy}$ denotes the 2D spatial Fourier transform which operates on the real part of $S(x, y, \omega)$. $\tilde{I}\left(f_x, f_y\right) = \mathcal{F}_{xy}[I(x, y)]$, is the spatial Fourier transform of the static speckle pattern. Eq. (2) states that $\tilde{S}\left(f_x, f_y, \omega\right)$ is given by the convolution of $\tilde{I}(f_x, f_y)$ and Dirac δ function shifted along f_x axis by $\Delta f_x = \omega/2\pi v$.

As can be seen in Fig. 4f, $\tilde{S}(f_x, f_y, \omega)$ consists of two copies of $\tilde{I}(f_x, f_y)$, one is displaced in the positive direction of f_x, while the other in the negative direction. Therefore, if the bandwidth of $\tilde{I}\left(f_x, f_y\right)$ is greater than the displacement Δf_x, it will be difficult to separate the two individual bands, meaning that the periodic modulation could no longer be resolved in the spectral map $S(x, y, \omega)$. As a consequence of the loss of the spectral resolutions in temporal- and spatial-frequency domains, the dynamic speckles degenerates into chaotic and uncorrelated fluctuations. In our case, the translational speckles are replaced by the boiling speckles when the moving speed increases. Because Δf_x is inversely proportional to the speed, the greater the speed is, the closer the two