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Reprint of : Floquet Majorana fermions in superconducting quantum dots

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HIGHLIGHTS

- We search for Majorana fermions in superconducting quantum dot arrays.
- Periodically variation of single-electron energies induces Floquet Majorana fermions.
- Floquet engineering allows us to tune their existence, localization and properties.

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ABSTRACT

We consider different configurations of ac driven quantum dots coupled to superconductor leads where Majorana fermions can exist as collective quasiparticles. The main goal is to tune the existence, localization and properties of these zero energy quasiparticles by means of periodically driven external gates. In particular, we analyze the relevance of the system and driving symmetry. We predict the existence of different sweet spots with Floquet Majorana fermions in configurations where they are not present in the undriven system.

1. Introduction

There are condensed matter systems which can hold collective quasiparticles that are their own antiparticles, therefore satisfying the Majorana condition [1–3]. These quasiparticles are termed Majorana Fermions (MFs) and follow non-abelian statistics. Detection of MFs in solid state systems has been recently experimentally proposed [4–6,53]. Recently, the interest in encoding a qubit in these kinds of excitations has grown due to the possibility to be non-local, a property which has a great potential in quantum computation due to the robustness of the qubit against local perturbations [7]. Furthermore, how to tune MFs in condensed matter systems is one of the main purposes of research in the emergent field of topological quantum computation.

In the last years, different works have shown how the application of ac fields enriches the properties of these quasiparticles and facilitate their tunability. For instance, it is possible to generate Floquet Majorana fermions (FMFs) as steady-states of non-equilibrium systems which present interesting properties for quantum computation: non-locality and non-abelian statistics [8,9].

In every system with particle–hole symmetry, the quasiparticles come in pairs $\gamma_{-E}^{\dagger} = \gamma_E$, therefore they can hold MFs as long as the energy can be tuned to zero. One of the simplest and most tunable system with particle hole symmetry is a double quantum dot (QD) connected via an s-wave superconductor [10]. It is well known that the proximity effect induces Cooper pairs correlations across the dots [11,12] generating effectively superconductivity [13]. Interestingly, fractional Josephson effect, a signature of the presence of MFs [5,14,15], in a quadruple quantum dot in the presence of an s-wave superconductor has been predicted by Markus Büttiker and coworkers [16].

The advantage that configurations of a few QDs connected to s-wave superconductors present, in order to generate and detect MFs, in comparison with nano-wires [17–19] or long QD chains [20–22] proposals is their great tunability, while in the latter the MFs have topological protection.

In this paper we analyze two different configurations of QDs in proximity to superconducting leads such that Cooper pair correlations are induced between the neighboring dots as long as the coherence length is larger than the distance between them. We include periodically driven gates and search for the conditions for

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appearance of FMFs. The paper is organized as follows: in Section 2 we present the model, in Section 3 we discuss the generation of FMFs in a double and a triple superconducting QD. Finally, we present our conclusions in Section 4.

2. Undriven system

Systems of QDs coupled to s-wave superconductors have been a subject of study [11,13,16] because the proximity effect induces Cooper pair correlations that can be easily detected due to the low number of degrees of freedom in QDs. In a system where neighboring QDs are coupled through superconducting reservoirs as in Fig. 1, in the limit of large superconducting gap the superconductors can be traced out and an effective Hamiltonian for the dots is obtained [23,24] as

$$H = \sum_{i,\sigma} \mu_{i,\sigma} d^{\dagger}_{i,\sigma} d_{i,\sigma} + \sum_{i,\sigma} (t_{i,i+1} d^{\dagger}_{i,\sigma} d_{i+1,\sigma} + \Delta_{i,i+1} d_{i,\sigma} d_{i+1,\bar{\sigma}} + h.\,c.), \tag{1}$$

which already contains effective superconductivity between neighboring dots. The fermionic operator $d_{i,\sigma}$ represents the annihilation of an electron in the i-QD with spin σ. The symbol $\bar{\sigma}$ means the opposite spin to σ, which can be $\sigma = \uparrow, \downarrow$. μ_i is the onsite energy in i-QD, the parameter $t_{i,i+1}$ is the effective tunneling probability from dot i to dot $i+1$ through the superconductor by virtual occupation of the above gap excitations and $\Delta_{i,i+1}$ is the effective superconducting amplitude due to the superconductor connecting the i and $i+1$ dots. If a large magnetic field is applied to the dots only one spin comes into play. However, the magnetic fields have to be non-collinear in order to have s-wave type Cooper pair correlations (see Fig. 1) [10]. In this configuration, it is more natural to work on the basis of the quantization axes given by the magnetic field in each dot. For that purpose, we have to perform the rotation

$$d_{2,\sigma} \to \cos\frac{\theta}{2} d_{2,\sigma} + \sigma \sin\frac{\theta}{2} d_{2,\bar{\sigma}} \tag{2}$$

as the magnetic field in the central QD forms an angle θ with the magnetic fields in the left and right QDs (see Fig. 1). The low-frequency Hamiltonian will be given by Eq. (1) by neglecting the contribution from the high-energy spin direction in each dot (keeping $\sigma = \downarrow$):

$$H = \sum_{i} \mu_i d^{\dagger}_i d_i + \sum_{i} \{ t'_{i,i+1} d^{\dagger}_i d_{i+1} + \Delta'_{i,i+1} d_i d_{i+1} + h.\,c.\}, \tag{3}$$

where $d_i \equiv d_{i,\downarrow}$, $t'_{i,i+1} \equiv t_{i,i+1}\cos(\theta/2)$ and $\Delta'_{i,i+1} \equiv \Delta_{i,i+1}\sin(\theta/2)$. Therefore the normal and superconducting tunneling amplitudes are renormalized and their renormalization depends on the angle between the magnetic field directions. This dependence introduces a simple way to tune externally the coupling parameters of the system [10].

In order to obtain the excitation spectrum of the system the Hamiltonian is written in the Nambu basis $\Psi = (d_1, d_1^{\dagger}, \ldots, \ldots, d_N, d_N^{\dagger})$ as

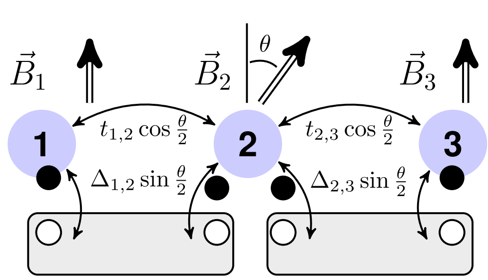

Fig. 1. Scheme of three QDs coupled by tunnel and coupled to a superconductor. The existence of Cooper pairs generates correlations of the type $d_{i,\sigma} d_{i+1,\bar{\sigma}}$ in the effective Hamiltonian for the QDs. The applied magnetic fields and their directions are also shown in the picture. The angle θ controls the ratio $\Delta_{i,i+1}/t_{i,i+1}$ (see text below).

$$H = \frac{1}{2}\Psi^{\dagger} h \Psi + \frac{1}{2}\sum_i \mu_i. \tag{4}$$

For a triple QD h reads

$$h = \begin{pmatrix} \mu_1 & 0 & t'_{1,2} & -\Delta'_{1,2} & 0 & 0 \\ 0 & -\mu_1 & \Delta'_{1,2} & -t'_{1,2} & 0 & 0 \\ t'_{1,2} & \Delta'_{1,2} & \mu_2 & 0 & t'_{2,3} & -\Delta'_{2,3} \\ -\Delta'_{1,2} & -t'_{1,2} & 0 & -\mu_2 & \Delta'_{2,3} & -t'_{2,3} \\ 0 & 0 & t'_{2,3} & \Delta'_{2,3} & \mu_3 & 0 \\ 0 & 0 & -\Delta'_{2,3} & -t'_{2,3} & 0 & -\mu_3 \end{pmatrix}. \tag{5}$$

The eigensystem of h ($h\mathbf{v}_i = \lambda_i \mathbf{v}_i$) determines the quasiparticles, given by $\gamma_i = \mathbf{v}_i \cdot \Psi$. A zero-energy solution, $\lambda_i = 0$, implies the presence of a pair of Majorana quasiparticles.

In the case of a double QD one can choose an angle such that $\Delta'_{1,2} = \pm t'_{1,2}$ and if $\mu_1 = 0$, there are two MFs given by

$$\gamma_1 = \frac{1}{\sqrt{2}}(d_1 \mp d_1^{\dagger}),\ \gamma_2 = \frac{1}{\sqrt{2}\sqrt{1+\delta^2}}\left\{(d_2 \pm d_2^{\dagger}) - \delta(d_1 \pm d_1^{\dagger})\right\}, \tag{6}$$

where $\delta = \mu_2/2t'_{1,2}$. Only in the case where $\mu_2 = 0$ the MFs are spatially separated [10]. In the case of a triple QD, assuming $\Delta'_{i,i+1} = \pm t'_{i,i+1}$ and $\mu_1 = 0$, there are two MFs given by

$$\gamma_1 = \frac{1}{\sqrt{2}}(d_1 \mp d_1^{\dagger}),\ \gamma_2 = \frac{(d_3 \pm d_3^{\dagger}) - \alpha(d_2 \pm d_2^{\dagger}) + \beta(d_1 \pm d_1^{\dagger})}{\sqrt{2}\sqrt{1+\alpha^2+\beta^2}}, \tag{7}$$

where $\alpha = \mu_3/2t'_{2,3}$ and $\beta = \mu_2\mu_3/4t'_{12}t'_{23}$. In the case where μ_2 or μ_3 are zero the MFs are spatially separated [25]. Interestingly, the manipulation of the onsite-energies allows us to change the localization of the MFs, which would be relevant for their detection in transport [10].

3. Floquet Majorana fermions

In the following, we will apply external ac fields in order to change periodically the onsite energies of the QDs and in this way obtain FMFs as steady-state solutions of the non-equilibrium problem.

For every system described by a time-periodic Hamiltonian a set of solutions exists, called Floquet states, which have the form $|\psi_n(t)\rangle = e^{-i\epsilon_n t}|u_n(t)\rangle$, where $|u_n(t)\rangle$ are time periodic functions called Floquet modes and ϵ_n are the so-called quasienergies [26–28]. As the quasienergies are only defined modulo Ω, where $\Omega = 2\pi/T$ and T is the period of the Hamiltonian, a system with particle–hole symmetry (with excitations in pairs $\gamma^{\dagger}_{-\epsilon} = \gamma_{\epsilon}$) will hold FMFs if $\epsilon = 0, \pm\Omega/2$. If the frequency is large enough, it is a good approximation to consider the time-averaged Hamiltonian to describe the dynamics. For lower frequencies, where multi-photon processes are relevant, the dynamics becomes more involved but there is also a way to find an effective time-independent Hamiltonian which includes as many photon processes as necessary [29,30].

The motivation to consider periodically driven quantum systems is the fact that their time-evolution is governed by an effective time-independent Hamiltonian, whose properties can be engineered according to the particular purposes. This method, called Floquet engineering, has been employed to achieve dynamic localization [31–33], photon-assisted tunneling [26,34] or nobel topological band structures [35–41,54,55].

The application of degenerate perturbation theory in the extended Floquet Hilbert space provides a high-frequency expansion (in powers of $1/\Omega$) for this effective Hamiltonian, such as $H_F = \sum_{\nu=0}^{\infty} H_F^{\nu}$ [29]. With the definition of the Fourier components of the time-periodic Hamiltonian