Physica E 78 (2016) 92–99

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Photon correlations generated by inelastic scattering in a one-dimensional waveguide coupled to three-level systems

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HIGHLIGHTS

- Study non-classical light in a waveguide coupled to three-level systems (3LS).
- Optimize parameters for strong correlations using total inelastically scattered flux.
- Show 3LS are better candidates for experimental study of photon–photon correlations.
- Show how slow light effect is expressed in correlations.
- Find two-photon wavefunction for (i) two 3LS far apart and (ii) one 3LS plus a mirror.

ARTICLE INFO

Article history:
Received 16 August 2015
Received in revised form
5 November 2015
Accepted 7 November 2015
Available online 10 November 2015

Keywords:
Quantum optics
Two-particle scattering
Electromagnetically induced transparency
Waveguide QED
Correlated photons
Non-classical light

ABSTRACT

We study photon correlations generated by scattering from three-level systems (3LS) in one dimension. The two systems studied are a 3LS in a semi-infinite waveguide (3LS plus a mirror) and two 3LS in an infinite waveguide (double 3LS). Our two-photon scattering approach naturally connects photon correlation effects with inelastically scattered photons; it corresponds to input–output theory in the weak-probe limit. At the resonance where electromagnetically induced transparency (EIT) occurs, we find that no photons are scattered inelastically and hence there are no induced correlations. Slightly away from EIT, the total inelastically scattered flux is large, being substantially enhanced due to the additional interference paths. This enhancement carries over to the two-photon correlation function, which exhibits non-classical behavior such as strong bunching with a very long time-scale. The long time scale originates from the slow-light effect associated with EIT.

1. Introduction

The many similarities between quantum transport of electrons (conduction) and optical phenomenon (propagation of EM radiation) have been used over the years to enrich both fields. While a scattering approach to the propagation of light, with input and output amplitudes, is quite natural in both classical and quantum optics [1,2], a comparable approach to electronic phenomena developed slowly. First introduced by Landauer [3,4], it was subsequently substantially developed by Büttiker [5–7]. This approach was then used, for instance, to develop parallels in mesoscopic physics between electronic and photonic phenomena, such as coherent backscattering of electrons or photons from disordered media [8,9]. Another example is in the development of semiclassical (or eikonal) approximations to quantum chaotic phenomena and the inclusion of diffractive effects [10]. While these parallels were developed mainly in the non-interacting-particle or linear-optics regime, interacting particles and the corresponding nonlinear regime are, of course, of key interest in both photonic and electronic transport. One particular setting that has received a great deal of attention in the quantum transport community is one-dimensional (1D) electrons interacting with local quantum impurities, a setting that includes for instance the Kondo problem, Anderson impurity model, and Bethe–Ansatz solutions [11–13]. The parallel photonic system is a one-dimensional EM waveguide strongly coupled to discrete non-linear quantum elements such as atoms, quantum dots, or qubits; in analogy with “cavity QED” [2,14], the study of such systems is known as “waveguide QED.”

The study of waveguide QED has increased rapidly over the past decade. Prior to that, there were a few early papers on the subject [15–19] that, for instance, exploited many-body approaches developed for electronic problems. The dramatic increase in interest starting in the period 2005–2008 [20–24] was driven by

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http://dx.doi.org/10.1016/j.physe.2015.11.004
1386-9477/

experimental progress toward achieving strong coupling between the waveguide and the local quantum system. Indeed, several experimental waveguide-QED platforms are being actively pursued. These include superconducting qubits coupled to a microwave transmission line [25–30], semiconductor quantum dots coupled to either a metallic nanostructure [31,32] or a photonic-crystal waveguide [33], and more traditional quantum optics settings in which atoms provide the local quantum system and the waveguide is an optical fiber or glass capillary [34,35]. Interesting waveguide-QED effects occur when the coupling to the waveguide dominates other emission or dephasing processes. Experiments in this interesting regime have been performed in several of the above waveguide-QED platforms.

Two aspects of waveguide QED have attracted particular attention theoretically: the manipulation of single photons and the production of non-classical light. In the single photon arena, a variety of devices have been proposed that build on the manipulation of single photons by qubits or three-level systems (3LS) that is possible in 1D systems; for representative work in this area see Refs. [34,33,36] and references therein. With regard to non-classical light, the main characteristics studied are the photon–photon correlation function (also called the second-order coherence [37]) and the photon statistics. The majority of work on these topics has treated a single quantum system coupled to the waveguide, where the single quantum system is modeled as a two-level system (2LS) or the only slightly more complicated driven 3LS (for very recent work along these lines see, for example, Refs. [38–41]). Correlation effects in a multi-qubit waveguide have been studied in a number of recent papers using a variety of techniques [42–55]. In most of these, the Markovian approximation is required in order to simplify the interactions between the qubits via the waveguide [42–48]. There are, however, a few non-Markovian results [49–55] which have been used to delineate the range of validity of the Markov approximation.

Here we extend our recent results on multiple 2LS waveguide QED [49–51] to the case in which driven 3LS are used. We calculate the two-photon wavefunction and focus on photon–photon correlations. We find that these correlations are substantially enhanced in systems containing 3LS, making them better experimental candidates for further study of the non-classical light produced. Furthermore, we find that the complexity of the structure enhances the photon–photon correlations—they are enhanced by adding additional nonlinear elements (qubits) as well as by simply adding a mirror. The photons can be either bunched or anti-bunched depending on the situation, and we find cases of both strong bunching and anti-bunching.

The paper is organized as follows. In the next section, we first recap the standard model of waveguide QED and a 3LS and summarize our approach to finding the two-photon wavefunction. Then we present the physical quantities that are calculated, emphasizing the total inelastic scattering as a measure of the correlated part of the wavefunction. Results for a single 3LS are presented in Section 3 as a basis for comparison to the more complex structures studied later. In Section 4 we add a mirror to the system, thus studying a single 3LS in a semi-infinite waveguide. Section 5 covers results for two 3LS in an infinite waveguide. In the results of both Sections 4 and 5, inelastic scattering is enhanced, suggesting more visible correlation effects. Finally, in Section 6 we discuss implications of the results and conclude.

2. Model and observables

2.1. Waveguide QED model with multiple three-level systems

The standard Hamiltonian of waveguide QED [20,22] consists of

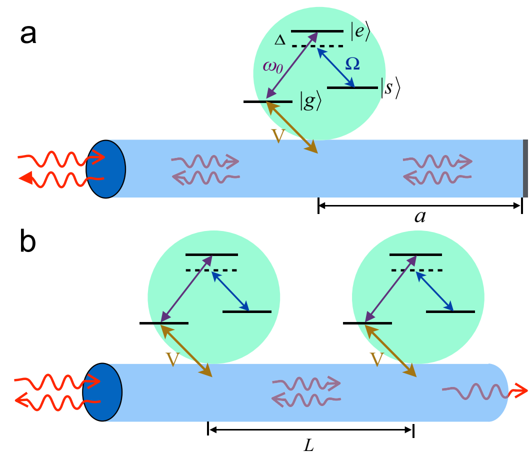

Fig. 1. Schematic of waveguide QED. (a) Single 3LS coupled to a semi-infinite waveguide with qubit-mirror separation a, $\omega_e = \omega_0$ and $\omega_s = \omega_0 - \Delta$. (b) Two identical 3LS, separated by distance L, coupled to an infinite waveguide.

a one-dimensional bosonic field that can travel to the left or right coupled to N local quantum systems, often called simply qubits. For a schematic see Fig. 1. Within the rotating-wave approximation, the Hamiltonian in real space reads (taking $\hbar = c = 1$)

$$H = H_{\rm QS} - i\int_{-\infty}^{\infty} dx\left[a_{\rm R}^{\dagger}(x)\frac{d}{dx}a_{\rm R}(x) - a_{\rm L}^{\dagger}(x)\frac{d}{dx}a_{\rm L}(x)\right] + \sum_{i=1}^{N}\sum_{\alpha={\rm L,R}} V_i\int_{-\infty}^{\infty} dx\,\delta(x-x_i)[a_{\alpha}^{\dagger}(x)\sigma_{ge}^{(i)} + \sigma_{eg}^{(i)}a_{\alpha}(x)], \tag{1}$$

where $\sigma_{eg}^{(i)} = |e\rangle_i\langle g|$ denotes the Pauli raising operator of the i-th qubit with position x_i and coupling strength V_i, and $a_{\rm R,L}$ denote the annihilation operators of right- or left-going photons. The corresponding decay rate of the i-th qubit to the waveguide is $\Gamma_i \equiv 2V_i^2$. Throughout this paper, the coupling of all of the qubits is the same, V. In order to assess the maximum possible non-classical light effects that could be present, we focus on the lossless limit.

The local quantum systems that we consider here are identical 3LS, $H_{\rm QS} = \sum_i H_{\rm 3LS}^{(i)}$. The Hamiltonian for a Λ-type 3LS is

$$H_{\rm 3LS} = \omega_0|e\rangle\langle e| + \omega_s|s\rangle\langle s| + \frac{\Omega}{2}(|e\rangle\langle s| + |s\rangle\langle e|), \tag{2}$$

in which Ω is the Rabi frequency of the classical driving and $\omega_s = \omega_e - \Delta$ with Δ being the detuning between the driving frequency and the frequency of the $|s\rangle$ to $|e\rangle$ transition. (The frequency corresponding to the ground state is taken to be zero.) Finally, we note that a mirror can be introduced as a boundary condition when solving for the single-photon wavefunction [51,56–58].

To construct the two-photon scattering wavefunction, we use the Lippmann–Schwinger equation [59,49–51], in which the Pauli raising and lowering operators, σ_{eg} and σ_{ge}, are replaced by bosonic creation and annihilation operators, $b^{\dagger}$ and b. (A similar approach has been used in the case of two-electron scattering [60,61].) To satisfy the level statistics, it is necessary to introduce an additional on-site repulsion U to be taken as infinite at the end. For a 2LS, it is known that this approach correctly gives all measurable quantities [49,51]. For a 3LS, in addition to repulsion for each upper level, an extra term has to be added so that the double occupancy can be fully ruled out: the repulsion operator $\tilde{V}$ is

$$\tilde{V} = \frac{U}{2}(b_e^{\dagger}b_e^{\dagger}b_eb_e + b_s^{\dagger}b_s^{\dagger}b_sb_s + 2b_e^{\dagger}b_eb_s^{\dagger}b_s). \tag{3}$$

Note that the coefficient of the last term is chosen for convenience; any coefficient would be canceled out after taking $U \to \infty$. Once a