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Wave propagation in viscoelastic single-walled carbon nanotubes with surface effect under magnetic field based on nonlocal strain gradient theory

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HIGHLIGHTS

- A model is derived for viscoelastic SWCNT with surface effect under magnetic field.
- Closed-form dispersion relation between phase velocity and wave number is derived.
- Size-dependent effect on the phase velocity is significant at high wave numbers.
- Phase velocity can increase by decreasing damping or increasing magnetic intensity.
- Damping ratio increases by increasing wave number or decreasing magnetic intensity.

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ABSTRACT

The governing equation of wave motion of viscoelastic SWCNTs (single-walled carbon nanotubes) with surface effect under magnetic field is formulated on the basis of the nonlocal strain gradient theory. Based on the formulated equation of wave motion, the closed-form dispersion relation between the wave frequency (or phase velocity) and the wave number is derived. It is found that the size-dependent effects on the phase velocity may be ignored at low wave numbers, however, is significant at high wave numbers. Phase velocity can increase by decreasing damping or increasing the intensity of magnetic field. The damping ratio considering surface effect is larger than that without considering surface effect. Damping ratio can increase by increasing damping, increasing wave number, or decreasing the intensity of magnetic field.

1. Introduction

Since carbon nanotubes (CNTs) are discovered in 1991 by Iijima et al. [1], worldwide commercial interest in CNTs have been reflecting in a production capacity [2] and therefore they have received huge attention. It has been reported by experimental investigations [3–10] that significant small-scaled effects on the mechanical and physical properties of small-scaled systems can be observed. Potential designs and applications of nano-systems involving CNTs rely on a realistic understanding of their mechanical and physical properties. Recently, an increasing interest can be observed on studying the vibration and wave propagation properties of CNTs under the magnetic field.

Some difficulties usually exist in experimental investigations due to extreme difficulty of controlling the precision of nano-scaled systems. To this end, molecular dynamics (MD) simulations and continuum mechanics based methods are often used for theoretical analysis and numerical simulations. Continuum mechanics based methods are of interest due to the face that MD simulations are usually complex and time consuming. Recently, the effects of magnetic field on the vibration or wave propagation in CNTs have been widely studied by some authors (see e.g., Refs. [11–35]). Most of these works are carried out based on Eringen's nonlocal elasticity theory [36]. In contrast to the classical elasticity theory, the stress of nonlocal elasticity theory at a reference point is supposed to depend not only on the strains at the reference point, but also on the strains at all other points in the body [36]. However, it was recently reported that the capability of identifying size-dependent stiffness for nonlocal elasticity theory may exist

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some limited problems [37–40]. To list a few, the stiffness enhancement effects observed from experimental investigations and as well as the modified couple stress (strain gradient) theory [41] cannot be predicted well by considering the nonlocal elasticity theory. And the nonlocal elastic model needs no extra boundary conditions to address the vibration problems and this is also questionable since some non-standard boundary conditions are expected to be added in these higher-order elasticity models [37,42,43]. To assess the true size-dependent effect on the structural responses, some authors studied the small-scaled structures using the unified nonlocal elasticity theory [44], the unified stress/strain gradient elasticity model [45–47] and the mixed nonlocal model [48,49]. More recently, in the context of thermodynamic framework, the nonlocal strain gradient theory [40] was presented to bring both of the length scales into a single theory. The stress of the nonlocal strain gradient theory accounts for not only the nongradient nonlocal elastic stress field [50], but also the non-locality of higher-order strain gradients stress field [51]. It was reported [40] that the nonlocal strain gradient theory can reasonably explain size-dependent phenomena and has been shown a good agreement with the results of MD simulations. Based on the nonlocal strain gradient theory, Li and Hu [52] examined the post-buckling analysis of size-dependent beams. Li et al. [53] studied the size-scaled effect on the wave propagation in functionally graded beams via the nonlocal strain gradient theory.

Recently, Xu et al. [54–56] reported that CNTs exhibit viscoelastic properties at the operational temperature range of $-196 \sim 1000\,^{\circ}$ C. More recently, the wave propagation analysis in viscoelastic SWCNTs (single-wall carbon nanotubes) is carried out by Pang et al. [57] and the effect of viscoelastic damping on the properties of wave propagation is discussed. In addition, it has been recently reported by many authors [58–66,57] that the surface effects play a very important role in determining the mechanical and physical properties of CNTs due to the very large surface-to-volume ratio of CNTs.

From the literature mentioned above, it is evident that there are strong scientific requirements to develop a good analytic model for the wave propagation analysis of viscoelastic SWCNTs with surface effect so that the true size-dependent effect on the wave propagation in viscoelastic SWCNTs can be assessed. The main motivation for this study is to develop an analytic model for the wave propagation analysis of viscoelastic SWCNTs with surface effect under magnetic field by using the nonlocal strain gradient theory, in which the stress accounts for not only the nonlocal elastic stress field but also the strain gradients stress field. In this paper, firstly, a viscoelastic constitutive equation is derived based on the nonlocal strain gradient theory. Then a derivation of the equation of wave motion is given and the wave propagation problem will be formed. Next, the dispersion relation between the phase velocity and the wave number is investigated. Finally, most important conclusions are summarized.

2. Viscoelastic constitutive equation based on nonlocal strain gradient theory

According to the nonlocal strain gradient theory, the total stress tensor $\boldsymbol{t}$ can be defined as [40]

$$\boldsymbol{t} = \boldsymbol{\sigma} - \nabla\boldsymbol{\sigma}^{(1)} \tag{1}$$

where the classical stress tensor $\boldsymbol{\sigma}$ (a work conjugate of the classical strain tensor $\boldsymbol{\varepsilon}$) and the higher-order stress tensor $\boldsymbol{\sigma}^{(1)}$ (a work conjugate of strain gradient tensor $\nabla\boldsymbol{\varepsilon}$) are defined as

$$\boldsymbol{\sigma} = \int_V \alpha_0(\boldsymbol{x}', \boldsymbol{x}, e_0 a)\boldsymbol{C}:\, \boldsymbol{\varepsilon}'(\boldsymbol{x}')\, \mathrm{d}V$$

$$\boldsymbol{\sigma}^{(1)} = l^2 \int_V \alpha_1(\boldsymbol{x}', \boldsymbol{x}, e_1 a)\boldsymbol{C}:\, \nabla\boldsymbol{\varepsilon}'(\boldsymbol{x}')\, \mathrm{d}V$$

Here e_0a and e_1a are nonlocal parameters (some works have been focused on identifying and incorporating the nonlocal parameter e_0a for nano-scale structures, see, e.g., [67,68]), l is the material length scale parameter (some studies on identifying the material length scale parameter l may refer to [41,69,70]). Suppose that the nonlocal functions $\alpha_0(\boldsymbol{x}, \boldsymbol{x}', e_0a)$ and $\alpha_1(\boldsymbol{x}, \boldsymbol{x}', e_1a)$ satisfy the condition [36], which assumes the linear nonlocal differential operator as the following formula:

$$\mathbb{L}_i = 1 - (e_i a)^2\nabla^2 \quad \text{for } i = 0, 1 \tag{2}$$

In the special case of Euler–Bernoulli and Rayleigh beams, by applying Eq. (2) into Eq. (1), a more general constitutive equation in a differential form can be obtained as the following form [40]:

$$[1 - (e_1a)^2\nabla^2][1 - (e_0a)^2\nabla^2]t_{xx} = E[1 - (e_1a)^2\nabla^2]\varepsilon_{xx} - El^2[1 - (e_0a)^2\nabla^2]\nabla^2\varepsilon_{xx}$$

where t_{xx} is the normal stress, ε_{xx} is the normal strain and $\nabla = \frac{\partial}{\partial x}$. Following the assumptions in [40], that is, retaining terms of order $O(\nabla^2)$ and assuming $e = e_0 = e_1$, the general constitutive can be simplified as

$$[1 - (ea)^2\nabla^2]t_{xx} = E(1 - l^2\nabla^2)\varepsilon_{xx} \tag{3}$$

Here E denotes Young's modulus. It is also known as the unified nonlocal elasticity theory [44], the unified stress/strain gradient elasticity model [45–47] or the mixed nonlocal model [48,49].

According to the Kelvin–Voigt viscoelastic damping model, the constitutive model (3) can be extent as the viscoelastic constitutive model

$$[1 - (ea)^2\nabla^2]t_{xx} = E(1 + \tau_d\frac{\partial}{\partial t})(1 - l^2\nabla^2)\varepsilon_{xx} \tag{4}$$

where τ_d is the damping coefficient. Next, the governing equation for viscoelastic SWCNTs with the surface effect under magnetic field will be derived based on the general viscoelastic constitutive model (4).

3. Equation of wave motion

The viscoelastic SWCNTs under magnetic field with the steady flexural waves propagating along the x-direction can be seen in Fig. 1. When flexural wave propagates in the SWCNTs, the equation of wave motion is perpendicular to the x-axis and it takes the following relationship based on the theory of Euler–Bernoulli beam with rotary inertia (or known as the Rayleigh beam) [71]

$$\frac{\partial Q}{\partial x} = \rho A\frac{\partial^2 w}{\partial t^2} - \rho I\frac{\partial^4 w}{\partial x^2\partial t^2} - F \tag{5}$$

$$Q = \frac{\partial M}{\partial x} \tag{6}$$

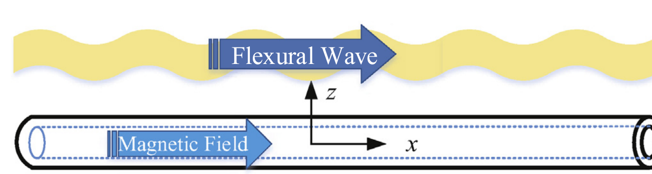

Fig. 1. Flexural wave propagation in viscoelastic SWCNTs under magnetic field.