Physica E 62 (2014) 1–9

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

A gradient model for Timoshenko nanobeams

Francesco Marotti de Sciarra *, Raffaele Barretta

Department of Structures for Engineering and Architecture, via Claudio 25, 80121 Naples, Italy

ARTICLE INFO

Article history:
Received 5 February 2014
Accepted 7 April 2014
Available online 15 April 2014

Keywords:
Timoshenko nanobeam
Size effect
Nonlocal thermodynamics
Nonlocal theory
Analytical solution

ABSTRACT

The elastostatic problem of a Timoshenko nanobeam is formulated by a new constitutive behaviour of gradient-type. Unlike previous approaches which directly substitute the expression of the nonlocal stress into the classical equilibrium equations, the proposed model starts from a nonlocal thermodynamic formulation. A suitable definition of the expression for the internal energy provides the variational formulation of Timoshenko nanobeams in terms of rotations and transverse displacements so that a higher-order system of ordinary differential equations is consistently obtained with the corresponding boundary conditions. Different from other nonlocal models, the solutions indicate that the stiffness of nanobeams is significantly increased at smaller scales due to size effects. The solutions corresponding to local models are perfectly obtained as a special case of the nonlocal ones.

1. Introduction

The mechanical behaviour of micro- and nano-sized components is significant different from the corresponding performance at larger scale due to size effects which are lacking at macroscale. Films, micro- and nano-beams and plates are commonly used in Micro Electro-Mechanical Systems (MEMS) and Nano Electro-Mechanical Systems (NEMS) devices or in nanocomposites as filling components for material reinforcements and experimental results show that such components exhibit size effects, see e.g. Hutchinson [1], Miller and Shenoy [2], Lam et al. [3], Yang [4], Guz et al. [5].

The small size of the samples requires a high precision so that the experiments are difficult to conduct and they often provide significant varying measurements, see e.g. the Young modulus for CNT [6,7].

Hence an in-depth understanding of analytical models and numerical studies of nano-structures is of utmost importance in design of nanomechanical systems.

It is well-known that size-effects cannot be exhibited by models based on classical continuum mechanics due to the lack of the material internal length. Accordingly atomistic models or new continuum theories can be introduced.

Atomistic/molecular dynamic simulations are computationally expensive and could become too complex to simulate nanostructures whereas high-order continuum models provide simple and effective tools in studying structural nanomodels [8–11].

In order to simulate interactions at the nanoscale, high-order continuum models can be used such as strain gradient theories [12,13], modified coupled stress theories [14–16] and nonlocal elasticity theories [17–21]. A nonlocal model for nanoroads under axial loads has been presented in Ref. [22] and a nonlocal FE approach in Refs. [23,24].

Since [25] most of the studies on micro- and nanobeams based on the Eringen nonlocal elasticity theory follow a similar approach, i.e. the stress tensor in the classical equilibrium equations is replaced by the corresponding nonlocal quantity (see e.g. Refs. [26–29]). As a result an increasing of the nonlocal parameter provides a reduction of the nanostructural stiffness so that the bending deflection of a simply supported beam under a distributed uniform load growths with respect to the corresponding local beam [30]. The opposite behaviour is revealed by a nanocantilever beam under a distributed uniform load since an increasing of the nonlocal parameter provides an increasing of the nanostructural stiffness. Note that experiments seem to show that the stiffness of structures tends to increase at smaller scales due to the size effects, see e.g. Ref. [3] with reference to epoxy polymeric beam.

In this paper the nonlocal elasticity theory is adopted to propose a new model for bending of Timoshenko nanobeams starting from a nonlocal thermodynamic framework (see e.g. Refs. [31,32]) which has been recently proposed to consistently obtain nonlocal formulations by suitably defining the internal energy or the Helmholtz free energy.

This new approach is used to derive the variational formulation associated with the Timoshenko nanobeam for the first time. Two nonlocal nanoscale parameters are considered: the former is linked to the normal strain and the latter is related to the transversal shear strain. Local and nonlocal stress resultants (bending moments and shear forces) are suitably defined. An explicit expression of the

* Corresponding author.
E-mail addresses: marotti@unina.it (F. Marotti de Sciarra), rabarret@unina.it (R. Barretta).

http://dx.doi.org/10.1016/j.physe.2014.04.005
1386-9477/

classical bending moment and shear force is provided. It is then shown in Appendix A that such quantities are related to the applied loads via the classical equilibrium equations.

The nonlocal Timoshenko model is then reformulated by using Green's formula and a system of two nonlocal fourth-order differential equations, in terms of the transversal displacement and rotation, is obtained together with the consistent higher-order boundary conditions containing the nonlocal nanoscale parameters. The structural nanostructural problem can be exactly solved thus providing the exact bending solution in terms of displacement, rotation and stress resultants for Timoshenko nanobeams.

The proposed exact nonlocal Timoshenko model reduces to the Euler–Bernoulli nonlocal model reported in Ref. [28] as a special case. Further, if the small-scale parameters vanish, the nonlocal Timoshenko model tends to the classical one.

For illustration purpose of the solution procedure, a cantilever nanobeam with a point load at the tip end and a simply supported nanobeam subjected to a uniform distributed load are considered. Both cases show that the nanostructural stiffness rises at the nanoscale with respect to the classical one and the solutions corresponding to the local models are faultlessly recovered.

2. A nonlocal thermodynamic framework for nanobeams

In nonlocal elasticity, the elastic strain tensor $\boldsymbol{\varepsilon}$ at a point $\mathbf{x}$ of the body Ω induces a stress tensor $\overline{\boldsymbol{\sigma}}$ not only at $\mathbf{x}$ but also at other points in the body and the stress has a decreasing magnitude with the distance.

The internal energy density e in a nonlocal elastic material can then be assumed in the form

$$e = e(\boldsymbol{\varepsilon}, R_1\boldsymbol{\varepsilon}, R_2\boldsymbol{\varepsilon}, \ldots, R_n\boldsymbol{\varepsilon}, s) \tag{1}$$

where $R_i\boldsymbol{\varepsilon}$ $(i=1,\ldots,n)$ are additional variables defined as functionals of the strain tensor $\boldsymbol{\varepsilon}$ and s is the entropy. In particular, the value of the internal variables $R_i\boldsymbol{\varepsilon}$ at a point $\mathbf{x}$ is related to the values of the strain over a neighbouring region of the body.

In a gradient elastic theory, the governing equations for a Timoshenko nanobeam can be obtained by a thermodynamic approach which accounts for the material nonlocality based on the relation (1).

Let us assume that the absolute temperature T is constant, i.e. there is no heat input due to radiation or conduction, and the density of mass is constant. The first law of thermodynamics for isothermal processes and for a nonlocal behaviour, see e.g. Refs. [33,34], can be formulated as follows:

$$\int_\Omega \dot{e}(\boldsymbol{\varepsilon}, \nabla\boldsymbol{\varepsilon}, \ldots, \nabla^n\boldsymbol{\varepsilon}, s)\, dV = \int_\Omega \overline{\boldsymbol{\sigma}} * \dot{\boldsymbol{\varepsilon}}\, dV \tag{2}$$

where the internal energy density e depends on strains $\boldsymbol{\varepsilon}, \nabla\boldsymbol{\varepsilon}, \ldots, \nabla^n\boldsymbol{\varepsilon}$ and entropy s and $\overline{\boldsymbol{\sigma}}$ is the nonlocal stress. The dot-superscript convention indicates the differentiation with respect to the time variable and the symbol ∇^n is the nth-order gradient. The energy balance in Eq. (2) can be written pointwise in Ω in the form

$$\dot{e} = \overline{\boldsymbol{\sigma}} * \dot{\boldsymbol{\varepsilon}} + P \tag{3}$$

where the additional thermodynamic scalar variable P is the nonlocal residual function and takes into account the energy exchanges between neighbour particles [35]. Since nonlocal effects due to elastic deformations are confined into the body, the residual P fulfils the insulation condition

$$\int_\Omega P\, dV = 0. \tag{4}$$

The second principle of thermodynamics for isothermal processes, in the nonlocal context, is written in its local form $\dot{s}T \geq 0$ everywhere in Ω where $\dot{s}$ is the internal entropy production rate per unit volume. In fact if the second principle holds in the global form $\int_\Omega \dot{s}T\, dV \geq 0$, there would be a class of deformation mechanisms which are reversible at the global level, since the second principle in the global form is equal to zero, but the same deformation mechanism turns out to be irreversible at the local level which is not physically acceptable.

Denoting by $\psi(\boldsymbol{\varepsilon}, \nabla\boldsymbol{\varepsilon}, \ldots, \nabla^n\boldsymbol{\varepsilon}, T)$ the Helmholtz free energy defined by means of the Legendre transform $\psi = e - sT$ and performing the time derivative of the Helmholtz free energy in connection with the second principle, the dissipation D at a given point of the body follows from the relation (3) in the form

$$D = \dot{s}T = \overline{\boldsymbol{\sigma}} * \dot{\boldsymbol{\varepsilon}} - \dot{\psi} + P \geq 0, \tag{5}$$

with $\dot{T} = 0$. The relation (5) represents the nonlocal Clausius–Duhem inequality for isothermal processes and the presence of the nonlocal residual function P guarantees the non-negativeness of the dissipation and accounts for material nonlocality. The body energy dissipation $\mathcal{E}$ follows by integrating the relation (5) to get

$$\mathcal{E} = \int_\Omega \dot{s}T\, dV = \int_\Omega \overline{\boldsymbol{\sigma}} * \dot{\boldsymbol{\varepsilon}}\, dV - \int_\Omega \dot{\psi}\, dV \geq 0. \tag{6}$$

Expanding the relation (6) and following the arguments used in Ref. [36], the inequality (5) becomes an equality so that the dissipation (5) is pointwise vanishing according to the reversible nature of the model. As a consequence, for any admissible deformation mechanism, it results

$$\int_\Omega \overline{\boldsymbol{\sigma}} * \dot{\boldsymbol{\varepsilon}}\, dV = \int_\Omega [d_{\boldsymbol{\varepsilon}}\psi * \dot{\boldsymbol{\varepsilon}} + d_{\nabla\boldsymbol{\varepsilon}}\psi * \nabla\dot{\boldsymbol{\varepsilon}} + \cdots + d_{\nabla^n\boldsymbol{\varepsilon}}\psi * \nabla^n\dot{\boldsymbol{\varepsilon}}]\, dV. \tag{7}$$

In what follows, the nonlocal model for Timoshenko nanobeams is developed starting from this nonlocal thermodynamic framework.

3. Timoshenko nanobeam kinematics

Let us consider a homogeneous isotropic nanobeam of length L as shown in Fig. 1. The x-coordinate is taken along the length of the beam, the y-coordinate along the thickness and the z-coordinate is taken along the width of the beam. The geometry and the applied loads of the nanobeam are such that the displacements (s_x, s_y, s_z) along the axes (x, y, z) are functions of the x- and y-coordinates. It is further assumed that the displacement s_z is identically zero and that the nonlocal behaviour is negligible in the thickness direction. The cross-sectional area A and the second moment of area I about the z-axis are

$$(A, I) = \int_A (1, y^2)\, dA. \tag{8}$$

The proposed Timoshenko nanobeam model (TNM) is based on the following displacement field [37]:

$$s_x(x,y) = -\varphi(x)y, \quad s_y(x,y) = v(x), \quad s_z(x,y) = 0 \tag{9}$$

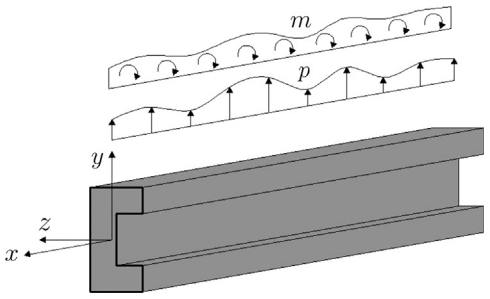

Fig. 1. Geometry and loading of the nanobeam.