Computational Materials Science 120 (2016) 60–69

Contents lists available at ScienceDirect

Computational Materials Science

journal homepage: www.elsevier.com/locate/commatsci

Improved models of solid foams based on soap froth

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ARTICLE INFO

Article history:
Received 22 July 2015
Received in revised form 11 February 2016
Accepted 19 March 2016
Available online 26 April 2016

Keywords:
Edge-length distribution
Laguerre tessellation
Model fitting
Power Voronoi tessellation
Relaxation
Stochastic foam model

ABSTRACT

Random Laguerre tessellations are widely used to model the cell systems of solid foams because they enable the reproduction of the cell-volume distributions found in real materials. However, the tessellations have planar cell faces, which over constrain the orientations of cell edges and adversely affect the distribution of edge lengths. The Surface Evolver was used to compute equilibrium microstructures of soap froth by minimizing the total surface area of Laguerre tessellations while preserving their cell-volume distributions. The resulting soap froth structures are more realistic than Laguerre tessellations because the edges of a face do not lie in the same plane and there are significantly fewer short edges. The edge-length distributions of soap froth models are in better agreement with experimental measurements of solid foams with open cells than those with closed cells. We argue that surface tension forces, which control area minimization, are more important during the formation of open-cell foams while viscous forces are more important during the formation of closed-cell foams but are not accounted for in existing models of foam formation. The distributions of other geometric characteristics, such as cell surface area, cell diameter, number of faces per cell and number of edges per face, are not adversely affected by area minimization.

1. Introduction

Random Laguerre tessellations – a generalization of the well-known Voronoi tessellation – have proven to be practicable and versatile models for the cell structure of solid foams with open and closed cells [15,25,27]. While the cells of Voronoi tessellations are generated from random seed points only, the generators of Laguerre tessellations also carry positive weights that can be interpreted as the radii of spheres centered at the seed points. To fit a Laguerre tessellation to a foam sample, the cell generators have to be chosen so that the distributions of foam characteristics are reproduced in the model. The cell system of real foams is usually highly regular so the same should be true for the models; consequently, tessellations generated by random systems of non-overlapping spheres are the most suitable choice. The packings can be simulated by random sequential adsorption (RSA, [6, p. 132]), dense packing algorithms like the force-biased algorithm [1], or molecular dynamics [11–13]. Besides being highly regular, these models have the advantage that each Laguerre cell completely contains the generating sphere. Consequently, the volume distribution of the cells is strongly related to the volume distribution of the generating spheres.

The tessellations have to be studied by simulation because their geometric characteristics are not analytically related to the parameters of the generators. In [18], an automatic fitting procedure was proposed for a Laguerre tessellation generated by a force-biased system of non-overlapping spheres with the two free parameters c and V_V – the coefficient of variation of the volume distribution of the spheres and the volume fraction of the sphere packing, respectively. The best model is found by minimizing a distance function between the moments of cell characteristics of the real foam sample and the model. More precisely, the first two moments of volume, surface area, mean width, and number of faces per cell were used in [18]. The possibility of including dihedral and interior angles in the model fitting is discussed in [25]; however, the analysis of samples of metallic and polymeric, open- and closed-cell foams shows that including the moments of the angles does not significantly improve the model fit.

It is known that each normal tessellation (meaning that four cells meet in a vertex as also required in Plateau's laws) with convex cells can be represented by a Laguerre tessellation [14]. So if one is interested in a random tessellation model with convex cells – which is particularly easy from a computational point of view – Laguerre tessellations are the natural choice. However, planar cell faces – as required by convexity of the cells – are not typical of real foams. This is best illustrated by the equilibrium structure of soap

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http://dx.doi.org/10.1016/j.commatsci.2016.03.029
0927-0256/

froth, which must obey Plateau's laws to minimize surface area. But no cell face can be a flat polygon with straight edges and satisfy the Plateau condition that all edges meet at the tetrahedral angle. Planar faces obviously restrict the shape and orientation of cell edges, but more important, they significantly influence the distribution of edge lengths [9–11].

Laguerre tessellations have a lot of short edges, but the edge-length distributions in soap froth are Gaussian-like and have relatively few short edges. This has also been observed in solid foams with open cells [16] even though, strictly speaking, they are not subject to Plateau's laws. Their realistic edge-length distributions have motivated the use of soap froth models as templates for developing finite-element models for the mechanics of random open-cell foams [4,5,7–10,17].

Beginning with the first studies of mechanical properties of three-dimensional open-cell foams [20,23], the short edges in Voronoi tessellations below some threshold length have been removed from the structure, artificially creating joints with more than four edges. It is argued that this procedure reduces computing time and has minimal effect on mechanical properties. These trimming processes are essentially unnecessary when soap-froth models are used as templates to create finite-element models of random foams because short edges are relatively rare [4,5,7,8,17].

Fig. 1 displays the edge-length distributions of several realizations of random Laguerre tessellations generated by force-biased sphere packings with varying parameters. The shape of the curves differs from typical edge-length distributions of real foams as shown, for example, in Fig. 4. In particular, a rather large number of short edges occur in all of the curves in Fig. 1. This implies that changing model parameters cannot reduce the number of short edges significantly.

Surface Evolver models of random soap froth are the most realistic physics-based models of a real foam available [11,12]. A more complete model of the structure formation process for solid foams would have to consider the expansion of a random suspension of gas bubbles to form the polyhedral cells in low-density foam, and include the viscous flow of the suspending fluid, which eventually solidifies to form the solid phase of the foam. But such models are unavailable. Therefore, to improve the fit of edge-length distributions, we use models of random polydisperse soap froth as templates for solid foams. This involves minimizing the surface area of fitted Laguerre tessellations by using the Surface Evolver [2], an interactive computer program for modeling liquid surfaces shaped by various forces (surface tension) and constraints (cell volumes). The surface evolves toward minimal energy (surface area) by simulating the process of evolution by mean curvature. Our goal is to investigate how the geometry of the models is affected by relaxation and whether the relaxed models better fit the samples.

This paper is organized as follows. First, we review some methods for processing and analyzing tomographic images of foams. We explain how the geometric characteristics estimated from the image data can be used for fitting Laguerre tessellations to the observed foam structures. Finally, the idea of relaxing Laguerre tessellations by using the Surface Evolver is introduced. We apply these techniques to six samples of open- and closed-cell foams. We compare geometric characteristics of the fitted Laguerre tessellations and their relaxed counterparts to investigate the extent to which relaxation improves the model fit.

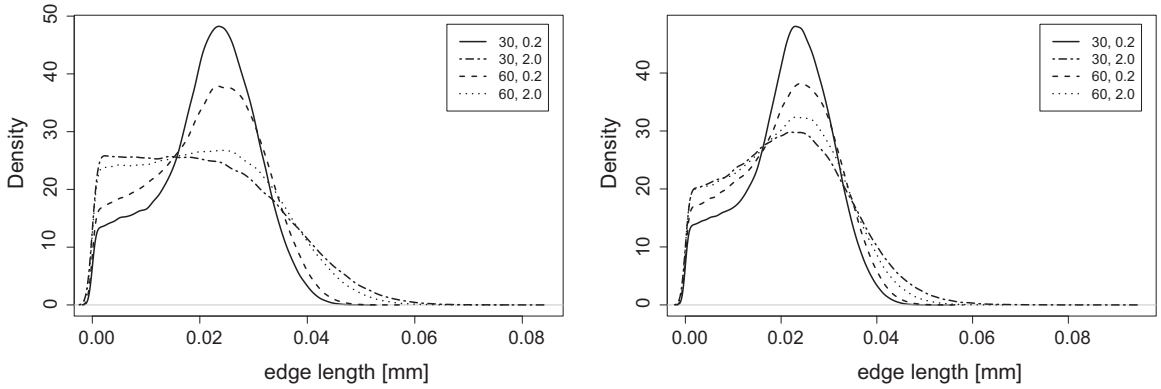
2. Processing and analysis of tomographic images

The microstructure of the real foam samples is typically imaged by micro computed tomography. In the resulting 3D images, the cells are usually connected, even in ideal closed-cell foams, due to insufficiently resolved cell walls and discretization effects. Therefore, the cells have to be separated in order to measure geometric characteristics of individual cells. This is achieved by using the following well established image processing chain:

1. Binarization yielding the strut system of the foam.
2. Euclidean distance transform on the pore space.
3. Inversion, i.e. each voxel gets as new gray value the difference between the maximal gray value and its old value.
4. Smoothing to remove superfluous local minima and avoid oversegmentation.
5. Watershed transform.

Here, avoiding oversegmentation is the crucial step as the watershed transform assigns a cell to each local minimum in the inverted distance image. Well-proven tools for removing superfluous minima are the h-minima transform, the height adaptive h-minima transform [21, Ch. 6] or a preflooded watershed transform [22]. For details on the choice of the method and its parameters see [19]. Sectional images of the reconstructed cell systems of the foam samples studied in Section 5 are shown in Fig. 2.

Based on the reconstructed cell systems, empirical distributions or moments of various cell characteristics can be estimated. Edge effects were taken care of by a minus sampling: a sub-window was chosen such that all cells centered inside it are completely observable. The sample was then restricted to these cells. All image



(a) Gamma (b) Lognormal

Fig. 1. Densities of the edge-length distribution in Laguerre tessellations generated by force-biased sphere packings with $V_V = 30\%,\ 60\%$ and $c = 0.2,\ 2.0$. Curves computed from simulations: five realizations with 10,000 cells for each parameter set in a window of 1 mm^3 with periodic edge treatment. Left: gamma volume distribution. Right: lognormal volume distribution.