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Theoretical investigation on a general class of 2D quasicrystals with the rectangular projection method

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ABSTRACT

We take a theoretical investigation on the reciprocal property of a class of 2D nonlinear photonic quasicrystal proposed by Lifshitz et al. in PRL 95, 133901 (2005). Using the rectangular projection method, the analytical expression for the Fourier spectrum of the quasicrystal structure is obtained explicitly. It is interesting to find that the result has a similar form to the corresponding expression of the well-known 1D Fibonacci lattice. In addition, we predict a further extension of the result to higher dimensions. This work is of practical importance for the photonic device design in nonlinear optical conversion progresses.

1. Introduction

In recent years, the 2D quasicrystals (2DQC) have become a focus in the structure design of nonlinear photonic crystals [1–6]. Some typical 2DQC structures such as Penrose tiles and polygonal quasicrystals with 8-, 10-, or 12-fold rotational symmetry have been researched in detail [7–9]. These 2DQC can be regarded as a natural extension of 1D quasiperiodic structures, which have made remarkable progresses in solving the multiple quasi-phase-matching problem in nonlinear optics [10–18]. Compared with 1D quasiperiodic structures, the 2DQC can provide multiple reciprocal vectors in a 2D plane simultaneously, which makes it possible to phase-match for both collinear and non-collinear optical conversion progresses. However, the reciprocal vectors of the 2DQC with a given symmetry are usually linearly correlated. Thus, it is not easy to employ them for an arbitrary nonlinear conversion progress. To solve this problem, research on the quasicrystal structure without rotational symmetry is necessary. Lifshitz et al. foresaw this and proposed a general method for the structure design of 2D nonlinear photonic quasicrystals [1]. In their work, a class of 2DQC without any rotational symmetry is constructed with a generalized dual-grid method and the obtained structure is capable of phase-matching for arbitrary 2D nonlinear optical conversion processes. The Fourier spectrum for the 2DQC structure can be also obtained with the dual-grid method, which is written as a complicated expression contains integrals and convolutions.

As is well-known that the projection method is commonly used to analyze the reciprocal property of quasicrystals [19], which is flexible to construct the quasicrystal structures as well. In addition, it enables us to obtain analytical solutions of both the position of reciprocal vectors and the related Fourier coefficients. In this paper, we found that the projection method might be more convenient to analyze the reciprocal property of the above 2DQC structure. With this method, the analytical expression for the Fourier spectrum of the structure is obtained, which is compatible with the result reported in Ref. [1] but with a simple and explicit form. Furthermore, we found that the resulting expression is similar to the corresponding expression of the well-known 1D Fibonacci lattice. This result is useful for the structure design and optimization in the application of nonlinear conversion processes [10,11,14].

2. Theoretical analysis

Let us start the analysis with a simple 1D quasicrystal, that is, the Fibonacci structure, which can be obtained by the projection method from a 2D square lattice to 1D with a special projection angle [19]. During the projection process, the projection area and the projection angle play an important role. The Fourier spectrum of 1D Fibonacci optical superlattice can be obtained with this

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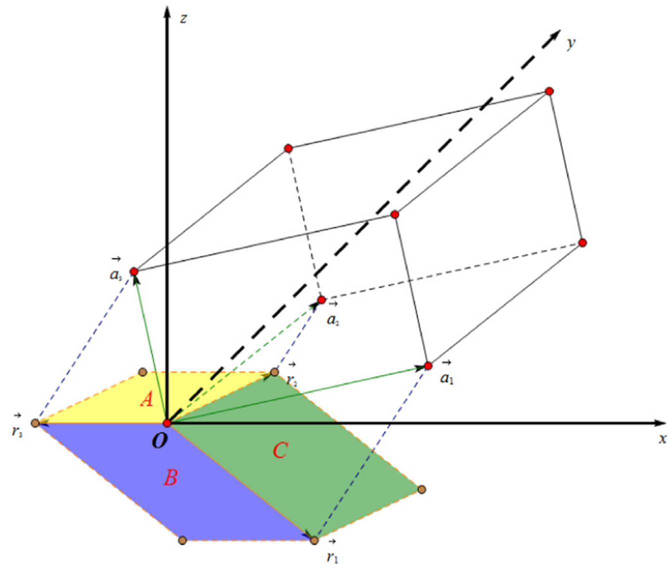

Fig. 1. (Color online) The schematic of the projection method to construct the 2DQC. $x-y$ plane is the projected plane, $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ are projections of $\vec{a}_1$, $\vec{a}_2$ and $\vec{a}_3$. The projected plane is constructed from three components A, B and C and each of them is a parallelogram.

method [16]:

$$g_{mn} = \frac{1+\tau}{D}\mathrm{sinc}\left[\pi(1+\tau)\left(\frac{mD_A + nD_B}{D}\right)\right] \quad (1)$$

where D_A and D_B are the widths of the two building blocks of the Fibonacci structure, $\tau = \frac{1+\sqrt{5}}{2}$, and $D = \tau D_A + D_B$.

The 2DQC structure constructed with the dual-grid method in Ref. [1] can also be obtained with the projection method, where a projection from the 3D cubic lattice to a 2D plane is needed. However, since the structure gets more complicated, it is easy to cause confusion when deducing the structural parameters with the cubic lattice. We found that it will be more convenient if a rectangular lattice is used instead, which is helpful for finding the unique relation of the projection parameters and the structural parameters of the quasicrystal structure.

To make it clear, we firstly choose a suitable projection window. As given in Fig. 1, the $x-y$ plane is set to be the projected plane, and $\vec{a}_1$, $\vec{a}_2$ and $\vec{a}_3$ are defined as the basic vectors in 3D rectangular lattice, whose projections to the $x-y$ plane are parallelogram vectors $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$. The projection area is an infinite plate perpendicular to the z axis. Its thickness can be given by:

$$d = a_1\cos\theta_1 + a_2\cos\theta_2 + a_3\cos\theta_3 \quad (2)$$

The projection angles θ_1, θ_2, S_2 are the angles between S_3, S_1, $\vec{a}_3$ and z axis, which depend on the structural parameters. We mainly focus on the projection result in $x-y$ plane, which is a 2D quasi-periodic lattice composed of several kinds of parallelograms with different shapes. It is clear from Fig. 1 that the projected plane can be constructed from three components A, B and C (which are yellow, blue and green respectively) and each of them is a parallelogram. They are supposed to be the fundamental components of the 2D quasicrystals. The areas of A, B, C (denoted as S_{23}, S_{12} and S_{31}, respectively) are important parameters used in the projection, which can be obtained by:

$$\begin{cases} S_{12} = a_1 a_2\cos\theta_3 \\ S_{23} = a_2 a_3\cos\theta_1 \\ S_{31} = a_3 a_1\cos\theta_2 \end{cases} \quad (3)$$

$$S = S_{23} + \tau_{12}S_{31} + \tau_{13}S_{12} \quad (4)$$

The parameter S in Eq. (4) is defined for convenience, which is related to the average area of the three components. In addition, the ratios of the three components are also needed to determine the structure, which can be expressed as follows while supposing that the number of A, B and C is $|A|_n$, $|B|_n$ and $|C|_n$ [17,18]:

$$\begin{cases} \tau_{12} = \lim\limits_{n\to\infty}\dfrac{|C|_n}{|A|_n} = \dfrac{a_2\cos\theta_2}{a_1\cos\theta_1} \\ \tau_{13} = \lim\limits_{n\to\infty}\dfrac{|B|_n}{|A|_n} = \dfrac{a_3\cos\theta_3}{a_1\cos\theta_1} \end{cases} \quad (5)$$

In order to give a general analysis, we choose a rectangular lattice represented by a set of distributed Dirac delta functions [18]:

$$u(\vec{r}) = \sum_{l,m,n}\delta(\vec{r} - \vec{r}_{lmn}) = \sum_{l,m,n}\delta(\vec{r} - l\vec{a}_1 - m\vec{a}_2 - n\vec{a}_3) \quad (6)$$

As the projection area is an infinite plate restricted by the thickness d, we can represent it by:

$$a(\vec{r}) = a(x, y, z) = \mathrm{rect}\left(\frac{z}{d}\right) \quad (7)$$

where the rect function is defined by:

$$\mathrm{rect}(x) = \begin{cases} 1, & |x| < 1/2 \\ 0, & elsewhere \end{cases}$$

With the lattice function $u(\vec{r})$ and the area function $a(\vec{r})$, we can define the position function $f(\vec{r})$, which represents the position of the lattices inside the projection area:

$$f(\vec{r}) = u(\vec{r}) \times a(\vec{r}) \quad (8)$$

According to convolution theorem, the Fourier transformation of $f(\vec{r})$ can be expressed as:

$$g^{3D}(\vec{G}) = U(\vec{G}) \otimes A(\vec{G}) \quad (9)$$

where $\otimes$ is the convolution operator, while $U(\vec{G})$ and $A(\vec{G})$ are the Fourier transform of the lattice function $u(\vec{r})$ and the area function $a(\vec{r})$, respectively, and both of them are known analytically. Thus, Eq. (9) can be simplified as:

$$g^{3D}(\vec{G}) = \frac{1}{a_1a_2a_3}\sum_{l,m,n}\frac{1}{\pi(G^z - G^z_{lmn})}\sin\left[\frac{1}{2}(G^z - G^z_{lmn})d\right] \delta(G^x - G^x_{lmn})\delta(G^y - G^y_{lmn}) \quad (10)$$

where $\vec{G}_{lmn}$ are the reciprocal lattice points and G^x_{lmn},G^y_{lmn}, G^z_{lmn} are components of x, y, z direction:

$$\vec{G}_{lmn} = l\vec{G}_1 + m\vec{G}_2 + n\vec{G}_3 = G^x_{lmn}\hat{x} + G^y_{lmn}\hat{y} + G^z_{lmn}\hat{z} \quad (11)$$

$\vec{G}_1$, $\vec{G}_2$ and $\vec{G}_3$ are three basic vectors of the reciprocal lattice, obeying the following orthogonality relation:

$$\vec{a}_i\cdot\vec{G}_j = 2\pi\delta_{ij}$$

Eq. (10) shows the Fourier transform of the projected lattices in 3D, to calculate the required Fourier transform of the 2DQC, we only need to set $G^z = 0$ and multiply the result by a factor of 2π due to the difference between 3D and 2D. Then the result can be written as:

$$\begin{aligned} g^{2D}(G^x, G^y) &= \frac{2\pi}{a_1a_2a_3}\sum_{l,m,n}\frac{1}{\pi G^z_{lmn}}\sin\left(\frac{1}{2}G^z_{lmn}d\right)\delta(G^x - G^x_{lmn})\delta(G^y - G^y_{lmn}) \\ &= \sum_{l,m,n} g^{2D}_{lmn}\delta(G^x - G^x_{lmn})\delta(G^y - G^y_{lmn}) \end{aligned} \quad (12)$$