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Spin conductivity of the two-dimensional ferroquadrupolar Heisenberg model

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ABSTRACT

We use the SU(3) Schwinger's boson theory to study the spin transport in the $S=1$ two-dimensional ferroquadrupolar Heisenberg model in the square lattice. We calculate the spin conductivity $\sigma(\omega)$ and analyzed the behavior of the AC and DC spin conductivities. The model presents a bilinear and biquadratic exchange interactions. Our results show that the system is an ideal spin conductor for $T>0$, because Drude's weight D_S (which represents the DC conductivity) is non zero for $T>0$ and the AC conductivity given by $\sigma^{reg}(\omega)$ tends to the infinity when $\omega \to 0$ which also correspond to the DC limit.

1. Introduction

Recently has there been an intense attention on the interplay of antiferromagnetism to high temperature super-conductivity [1] where a number of measurements were reported in a member of the pnictide family $BaFe_2(As_{1-x}P_x)_2$ as a function of the isovalent P-doping x. Among the most varied studies, the two-dimensional ferroquadrupolar Heisenberg model on a square lattice has been applied to the iron pnictides [2]. Different from quantum spin liquids, spin nematic states exhibit long-ranged quadrupolar order without conventional dipolar magnetic order [3]. The phase transitions in the two-dimensional anisotropic biquadratic Heisenberg model have been studied recently [4]. The biquadratic term arises from fourth-order perturbations in the exchange interaction and normally its value is small. As pointed out by Oitmaa and Hamer [5] the study of the Heisenberg antiferromagnet with biquadratic exchange interaction goes back to [6,7], where phases of both dipolar and quadrupolar nature were identified [3]. The traditional spin wave SU(2) theory is a very good approach for treating quantum spin models that have a magnetically (dipolar) ordered ground state. However this theory is not adequate to treat nematic, quadrupolar, octupolar, or higher multipolar ordering [3].

From an experimental point of view, recently has there been an intense research about the quantum Hall effect for spins and the magnon spintronics [8–12]. In these studies often only the sign differences between related quantities like magnetic fields and generated spin and charge currents are determined. The spin transport properties in the two-dimensional spin systems have been studied theoretically by Sentef et al. [13] who have analyzed the spin transport in the easy-axis Heisenberg antiferromagnetic model in two and three dimensions, at $T=0$. Damle and Sachdev [14] treated the two-dimensional case using the non-linear sigma model in the gapped phase. Pires and Lima [15–17] treated the two-dimensional easy plane Heisenberg antiferromagnetic model. Lima and Pires [18] studied the spin transport in the two-dimensional anisotropic XY model using the SU(3) Schwinger boson theory in the absence of impurities, Lima [19] studied the case of the Heisenberg antiferromagnetic model in two dimensions with Dzyaloshinskii–Moriya interaction. Zewei Chen et al. [20] analyzed the effect of spatial and spin anisotropy on spin conductivity for the $S=1/2$ Heisenberg model on a square lattice and more recently, Lima et al. [21] have studied the spin transport in the site diluted two-dimensional anisotropic Heisenberg model in the easy plane using the self-consistent harmonic approximation.

The aim of this paper is to study the spin transport of the two-dimensional ferroquadrupolar Heisenberg model on a square lattice with biquadratic exchange interaction using the SU(3) Schwinger boson theory which is also called as Bond Operator formalism. Recently, the dynamics structure factor was calculated for this model using this method in Reference [3].

The plan of this paper is as follows: In Section 2, we discuss the properties of the model. In Section 3 we present the SU(3) Schwinger boson formalism. In Section 4 we develop the Kubo formalism of the linear response to calculate the spin conductivity. In Section 5, we present the final remarks.

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2. The model

The $S=1$ two-dimensional ferroquadrupolar Heisenberg model on the square lattice is defined by the following Hamiltonian

$$\mathcal{H}=\sum_{\langle i,j\rangle}\left\{J_1(\mathbf{S}_i\times\mathbf{S}_j)+J_2(\mathbf{S}_i\times\mathbf{S}_j)^2\right\}. \tag{1}$$

where $\langle i,j\rangle$ stands for the sum over nearest-neighbor, J_1 is the exchange constant between the nearest-neighbor, J_2 is the intensity of the biquadratic term. We consider the value of spin $S=1$. The system is in the ferroquadrupolar phase, where the $J_1=J\cos\theta$ and $J_2=J\sin\theta$. The θ parameter controls the ratio of these two couplings. Recently, this model with $\sin\theta<0$ has been applied to the iron pnictides [2].

3. The Schwinger boson theory

The SU(3) Schwinger boson formalism has been derived for treat systems with single ion anisotropy for Papanicolau [22] being a generalization of the SU(2) formalism. In this formalism we choose the basis:

$$|x\rangle=\frac{1}{\sqrt{2}}(|1\rangle-|-1\rangle)),\quad |y\rangle=\frac{1}{\sqrt{2}}(|1\rangle+|-1\rangle)),\quad |z\rangle=-i|0\rangle \tag{2}$$

where $|n\rangle$ are eigenstates of S^z. This set of basis states respect the time-reversal invariance of the spin-nematic states [3]. The spin operators are then written via a set of three boson operators t_α, $(\alpha=x,y,z)$ defined by

$$t_x^\dagger|v\rangle=|x\rangle,\quad t_y^\dagger|v\rangle=|y\rangle,\quad t_z^\dagger|v\rangle=|z\rangle. \tag{3}$$

where $|v\rangle$ is the vacuum state. To force single site occupancy on each site we impose the constraint

$$t_x^\dagger t_x+t_y^\dagger t_y+t_z^\dagger t_z=1. \tag{4}$$

In terms of the t operators we can write

$$S^x=-i\left(t_y^\dagger t_z-t_z^\dagger t_y\right),\quad S^y=-i\left(t_z^\dagger t_x-t_x^\dagger t_z\right),\quad S^z=-i\left(t_x^\dagger t_y-t_y^\dagger t_x\right).$$

The states $t_x^\dagger|v\rangle$ and $t_y^\dagger|v\rangle$, both consist of $S^z=\pm1$ eigenstates and have the average $\langle S^z\rangle=0$. This property will preserve the disorder of the ground state.

In studying the disordered phases, such as the ferroquadrupolar phase, it is convenient to introduce two bosonic operators $u^\dagger$ and $d^\dagger$, given by [3]

$$u^\dagger=\tfrac{1}{\sqrt{2}}\left(t_x^\dagger+it_y\right),\quad d^\dagger=\tfrac{1}{\sqrt{2}}\left(t_x^\dagger-it_y\right), \tag{5}$$

and so

$$|1\rangle=u^\dagger|v\rangle,\quad |0\rangle=t_z^\dagger|v\rangle,\quad |-1\rangle=d^\dagger|v\rangle. \tag{6}$$

We have the constraint condition $u^\dagger u+d^\dagger d+t_z^\dagger t_z=1$. The spin operators can also be written as

$$S^+=\sqrt{2}\left(t_z^\dagger d+u^\dagger t_z\right),\quad S^-=\sqrt{2}\left(t_z^\dagger d+u^\dagger t_z\right),\quad S^z=u^\dagger u+d^\dagger d. \tag{7}$$

Schwinger's boson formalism is a mean field approximation that becomes accurate in the $N\to\infty$ limit. For the SU(3) Schwinger boson approach for spins $S=1$, the order parameter has eight components which correspond to the eight generators of the SU (3) group [3].

$$\mathcal{H}=\mathcal{H}_0+\mathcal{H}_1+\mathcal{H}_2, \tag{8}$$

where

$$\mathcal{H}_0=(1+t^4)J\sin\theta\frac{zN}{2}+\mu\sum_k\left(u_i^\dagger u_i+d_i^\dagger d_i+t^2-1\right), \tag{9}$$

$$\mathcal{H}_1=Jt^2\cos\theta\sum_{\langle ij\rangle}\left(u_i^\dagger u_j+d_i^\dagger d_j+h.c.\right)+J(\cos\theta-\sin\theta)t^2\sum_{\langle ij\rangle}\left(u_i^\dagger d_j^\dagger+d_i^\dagger u_m^\dagger\right), \tag{10}$$

$$\mathcal{H}_2=J\cos\theta\sum_{\langle ij\rangle}\left(u_i^\dagger u_i u_j^\dagger u_j+d_i^\dagger d_i d_j^\dagger d_j\right)-J(\cos\theta-\sin\theta)\sum_{\langle ij\rangle}\left(u_i^\dagger u_i d_j^\dagger d_j+d_i^\dagger d_i u_j^\dagger u_j\right)$$
$$+J\cos\theta\sum_{\langle ij\rangle}\left(u_i^\dagger d_j^\dagger u_j d_i+d_i^\dagger u_j^\dagger d_j u_i\right). \tag{11}$$

$\mu$is a temperature-dependent chemical potential with the local constraint $S_r^2=S(S+1)=2$. We obtain the mean field Hamiltonian making decoupling to the four operator terms

$$\mathcal{H}_2^{mf}=-2J(\cos\theta-\sin\theta)\sum_{\langle ij\rangle}\left[p\left(u_i^\dagger d_j^\dagger+d_i^\dagger u_j^\dagger\right)\right.$$
$$\left.+h.c.\right]+\frac{zN}{2}\left[J\cos\theta(1-t^2)^2+4J(\cos\theta-\sin\theta)p^2\right], \tag{12}$$

where $p=\langle u_i u_j\rangle$. This parameter is very small and can be neglected. The Hamiltonian can be written then in the form

$$\mathcal{H}=\frac{1}{2}\sum_k\psi_k^\dagger\mathbf{H}_{\alpha\alpha}\psi_k+E_0, \tag{13}$$

where $\psi_k^\dagger=\left(u_k^\dagger\ d_k^\dagger\ u_{-k}\ d_{-k}\right)$, and

$$\mathbf{H}_{\alpha\alpha}=\begin{pmatrix}\Lambda_k & 0 & 0 & \Lambda_k\\ 0 & \Lambda_k & \Lambda_k & 0\\ 0 & \Lambda_k & \Lambda_k & 0\\ \Lambda_k & 0 & 0 & \Lambda_k\end{pmatrix}, \tag{14}$$

$$\omega_k=\sqrt{\Lambda_k^2-\Delta_k^2}, \tag{15}$$

where

$$E_0=\frac{zN}{2}\left[J\cos\theta(1-t^2)^2+4J(\cos\theta+\sin\theta)p^2\right], \tag{16}$$

and

$$\Lambda_k=\lambda+4t^2\gamma_k\cos\theta, \tag{17}$$

$$\Delta_k=4t^2(\cos\theta+\sin\theta)\gamma_k, \tag{18}$$

$$\gamma_k=\frac{1}{2}[\cos(k_x)+\cos(k_y)]. \tag{19}$$

Once the excitation gap is null at $k=(\pi,\pi)$ we have the self-consistent equations

$$\lambda=-4t^2\sin\theta$$
$$t^2=2-\frac{1}{N}\sum_k\frac{\Lambda_k}{\omega_k}. \tag{20}$$

We use the SU(3) Schwinger's boson approach [22] to determine the regular part of the spin conductivity (AC conductivity) or continuum conductivity.

A spin current appears if there is a gradient of magnetic field $\nabla\vec{B}$, through the system. It plays the role of a chemical potential for spins. One can connect a low dimensional magnet with two bulk ferromagnetic. They act as reservoirs for spins [12,11]. One has a spin current flowing through the system if there is a difference, $\Delta\vec{B}$, between the magnetic fields at the two ends of the sample. As we are interested in calculating the longitudinal spin conductivity, we will add an external space and time-dependent magnetic field, $\vec{B}(x,t)$, applied along the $\hat{z}$ direction to the Hamiltonian equation (1).