Solid State Communications 181 (2014) 46–49

Contents lists available at ScienceDirect

Solid State Communications

journal homepage: www.elsevier.com/locate/ssc

Modeling of bi-equilibrium states in dielectric elastomer

Longgui Peng *

[a] Department of Applied Chemistry of Science College, Northwestern Polytechnical University, Xi'an, China
[b] Department of Materials Science and Engineering, Xi'an University of Science and Technology, Xi'an, China

ARTICLE INFO

Article history:
Received 29 September 2013
Received in revised form
28 November 2013
Accepted 28 November 2013
by J.F. Sadoc
Available online 7 December 2013

Keywords:
A. Polymer
B. Electromechanical loading
C. Phase transition
D. Instability

ABSTRACT

Dielectric elastomer is a soft active material, producing fast deformation under voltage-activation. Under a specific boundary condition, trussed dielectric elastomer elongates mimicking the behavior of biological muscle. During this process, dielectric elastomer experiences a snap from one deformation mode to another, though both at the electromechanical equilibrium states. Based on thermodynamics, models are established to investigate electromechanical coupling at the two equilibrium states. Particular emphasis is devoted to establishing the governing equations of the two deformation modes with physical interpretations. The transition of equilibrium state is discussed, to predict the attainable stable state for application.

1. Introduction

Dielectric elastomer is a soft polymer used as a dielectric insulator, usually in the shape of thin film, and is capable of generating reversible area expansion strain over 100% when subject to voltage [1]. Due to the compliance, dielectric elastomer features light weight, noiseless actuation and extreme stretchability. This feature resembles the performance of biological muscles, and applications have been explored in tunable lens [2], soft robots, etc. [3]. However, its low elastic modulus, of the orders of kPa, would result in an insufficient force feedback, causing the material susceptible to pull-in instability: a failure to withstand the excessive electrostatic stress in deformation [4,5]. Therefore, diverse routes are proposed to strengthen the dielectric elastomer in order to avert the instability, i.e., by introducing an extra polymer with a higher stiffness to achieve interpenetrating polymer networks [6,7], or by doping rigid ceramic of high permittivity [8,9].

Another effective method has been adopted recently in the actuator design, where the surface of the dielectric elastomer is trussed by stiff fibers in parallel. This configuration is inspired by the fish fin, as Fig. 1(a) shows [10]. With reference to related experiment; the trussed dielectric elastomer shows a quasi-linear strain in vertical direction, without pull-in. However, the performance of fiber-trussed dielectric elastomer (FTDE) varies greatly. For the same materials, the maximum strain changes from 35% [10,11] to 260% [12], while an interpretation on the difference has not been proposed. It is also observed that during elongation, the dielectric elastomer may jump to another deformation state, showing necking state, with loss of surface tension in the central area of the membrane [11].

In the current research, we proposed a model, based on thermodynamics, to develop the equations of state of the FTDE. A specific material model is employed to study the deformation and stability, and the results are discussed and concluded with respect to the corresponding experiments.

With reference to Fig. 1(b)–(c), considering a piece of dielectric elastomer sheet of original dimensions of L_1L_2H. Following the experimental procedures in Ref. [11], we first stretch the sheet horizontally to width of l_2 and maintain the prestretch by attaching stiff fibers. Consequently, the confinements generate lateral force of P' that keeps the membrane in tension. The fibers separate the surface into elementary unit segments of number N, with each area of $(L_1/N)l_2$. We then apply a battery with a voltage of Φ and mechanical load P to the FTDE, which deforms the current dimensions of l_1l_2h with accumulated charges $\pm Q$ on both surfaces. Note here, without losing generality, we first idealize the horizontal deformation is maintained at a constant stretch by P'. The assumption shall be discussed in the analysis that follows.

The battery, the load, and the elastomer with fibers constitute a thermodynamic system. During electromechanical deformation, when the in-plane dimensions increase by δl_1 and δl_2, the thickness reduces by δh, and the charge increase by δQ, so the free energy of the DE changes by δF and the free energy of the thermodynamics system changes by $\delta\Psi$. Thus

$$\delta\Psi = \delta F - P\delta l_1 - P'\delta l_2 - l_2\delta P' - \Phi\delta Q. \quad (1)$$

Define the specific Helmholtz free energy of the material by $\hat{W} = F/(L_1L_2H)$, stretches by $\lambda_1 = l_1/L_1$, $\lambda_2 = l_2/L_2$, and $\lambda_3 = h/H$,

* Corresponding author at: Department of Materials Science and Engineering, Xi'an University of Science and Technology, Xi'an, China. Tel.: +86 290 000 00.
E-mail address: penglonggui@163.com

0038-1098/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ssc.2013.11.033

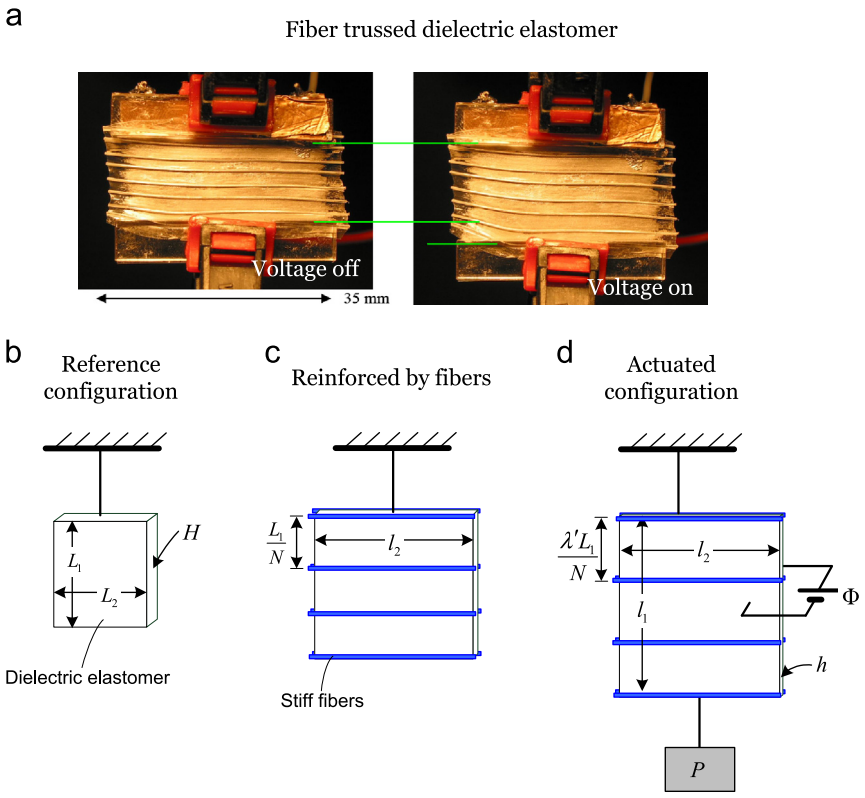

Fig. 1. (Color online) (a) A fiber trussed electric elastomer elongates linearly under voltage (Ref. [10]). According to experiments, we sketch the models as follows. The dielectric elastomer is prestretched first from (b) its original dimensions, with (c) stiff fibers to maintain the prestretch. (d) Subject to a voltage and mechanical load, the fiber trussed-dielectric elastomer (FTDE) deforms to the current dimensions.

stresses by $\sigma_1 = P/(l_2 h)$ and $\sigma_2 = P'/(l_1 h)$, electric field by $E = \Phi/h$, and electric displacement by $D = Q/(l_1 l_2)$. The stretch in each segment λ' is related to the overall vertical stretch as: $\lambda' = (\lambda_1 - \eta)/(1-\eta)$, with η being the fiber volume fraction of the dielectric elastomer in the reference state. Considering the fiber is electrically inactive and its volume fraction is insignificant [10–12], we reduce $\lambda' = \lambda_1$ and do not specifically distinguish the two variables in the investigation afterward.

The amount of charge on either surface relates to the electric displacement by $Q = Dl_1 l_2$, so the variation of the charge is

$$\delta Q = l_1 l_2 \delta D + D l_1 \delta l_2 + D l_2 \delta l_1. \tag{2}$$

The elastomer is taken to be incompressible, where $L_1 L_2 L_3 = l_1 l_2 l_3$ or $\lambda_1 \lambda_2 \lambda_3 = 1$. This enables us to regard λ_1 and λ_2 as independent variables, so that $\delta\lambda_3 = -\lambda_1^{-2}\lambda_2^{-1}\delta\lambda_1 - \lambda_2^{-2}\lambda_1^{-1}\delta\lambda_2$. Dividing both sides of Eq. (1) by the volume, $L_1 L_2 H$, and using Eq. (2), we obtain the free energy change in the thermodynamic system

$$\frac{\delta\Psi}{L_1 L_2 H} = \delta\hat{W} - (\sigma_1 + DE)\lambda_1^{-1}\delta\lambda_1 - (\sigma_2 + DE)\lambda_2^{-1}\delta\lambda_2 - E\delta D. \tag{3}$$

In this model, the Helmholtz free energy is taken to be a function of the three independent variables, $\hat{W}(\lambda_1, \lambda_2, D)$, as established in the nonlinear field theory [13]. So that

$$\frac{\delta\Psi}{L_1 L_2 H} = \left(\frac{\partial\hat{W}(\lambda_1,\lambda_2,D)}{\partial\lambda_1} - (\sigma_1 + DE)\lambda_1^{-1}\right)\delta\lambda_1 + \left(\frac{\partial\hat{W}(\lambda_1,\lambda_2,D)}{\partial\lambda_2} - (\sigma_2 + DE)\lambda_2^{-1}\right)\delta\lambda_2 + \left(\frac{\partial\hat{W}(\lambda_1,\lambda_2,D)}{\partial D} - E\right)\delta D. \tag{4}$$

Thermodynamics indicates that the system is in equilibrium only if the free energy change is minimum, $\delta\Psi = 0$, for any arbitrary and independent variations $\delta\lambda_1$, $\delta\lambda_2$ and δD. This law allows multi solutions, corresponding to the existence of different equilibrium states, even coexistence for two states. We first assume the horizontal stretch being constrained by the fibers at a constant level λ_2, that is $\delta\lambda_2 = 0$. Therefore the related production, the second item on the right hand side of Eq. (4) vanishes, implying the current free energy change is

$$\frac{\delta\Psi}{L_1 L_2 H} = \left(\frac{\partial\hat{W}(\lambda_1,\lambda_2,D)}{\partial\lambda_1} - (\sigma_1 + DE)\lambda_1^{-1}\right)\delta\lambda_1 + \left(\frac{\partial\hat{W}(\lambda_1,\lambda_2,D)}{\partial D} - E\right)\delta D. \tag{5}$$

Then we obtain the first set of equations for equilibrium state

$$\sigma_1 + ED = \lambda_1\frac{\partial\hat{W}(\lambda_1,\lambda_2,D)}{\partial\lambda_1}; \quad E = \frac{\partial\hat{W}(\lambda_1,\lambda_2,D)}{\partial D}. \tag{6}$$

We now employ a specific free energy function form for linear dielectrics with hyper-elasticity

$$\hat{W}(\lambda_1,\lambda_2,D) = -\frac{\mu}{2}\log\left(1 - \frac{\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2}\lambda_2^{-2} - 3}{J_m}\right) + \frac{1}{2\varepsilon}D^2. \tag{7}$$

The first item on the right hand side of Eq. (7) is the Gent strain energy model, with μ being the shear modulus and J_m being the extension limit [14]. The second item is the linear electrostatic energy in dielectric, with ε being the permittivity. Thus, the first set of equilibrium condition is specialized to the governing equation as

$$(\lambda_1\lambda_2)^2\left(\frac{\Phi}{H\sqrt{\mu/\varepsilon}}\right)^2 = \frac{\lambda_1^2 - \lambda_1^{-2}\lambda_2^{-2}}{1 - ((\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2}\lambda_2^{-2} - 3)/J_m)} - \lambda_1\frac{P}{\mu L_2 H}. \tag{8}$$

Eq. (8) suggests that the elastomer deforms in two directions: vertically and through the thickness, with the horizontal stretch remaining at the prestretch state. This mode of deformation is