CIRP Journal of Manufacturing Science and Technology 8 (2015) 22–33

Contents lists available at ScienceDirect

CIRP Journal of Manufacturing Science and Technology

journal homepage: www.elsevier.com/locate/cirpj

Modeling the influence of web thickness and length imperfections resulting from manufacturing processes on wound roll stresses

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ARTICLE INFO

Article history:
Available online 20 November 2014

Keywords:
Web
Thickness imperfection
Length imperfection
Winding simulation

ABSTRACT

Imperfection in thickness and length result from web manufacturing processes such as forming and coating. Webs are stored in wound roll form as they await subsequent processing. Herein an axisymmetric nonlinear winding model is developed. Results from this model will show how small imperfections in web thickness and length can lead to substantial residual stress variation in the wound roll. These stress variations can lead to partial or entire loss of the web in the wound roll. The results from the winding model can be used to determine what winding conditions can be used to mitigate the residual stresses. The results can also be used to determine practical limits for web imperfections to prevent residual stresses in the wound roll from producing material loss. Results from the model will be compared to test data on webs wound in the laboratory to verify the model and to generate an awareness of the magnitude of the problem.

Introduction

Webs of paper, plastic films, metals and other materials are formed by very different manufacturing processes. Paper is made by Fourdrinier forming, films are made by casting or orienting extruders and metal webs are produced in hot and cold rolling operations. Whatever formation process is used will induce some imperfection in the web in the form of thickness and length variation. As difficult as it is to form webs of uniform thickness that difficulty is matched when trying to produce a uniform coating thickness upon a web. Thus web thickness imperfections are oft the sum of formation and coating imperfection. Webs are defined as materials whose length greatly exceeds the width which greatly exceeds the thickness. Web forming and manufacturing processes are continuous and rate dependent and intermediate storage is required between process operations. Web thicknesses are typically small. This requires the web to be stored in wound roll form to prevent damage while awaiting subsequent processing or conversion to a final product. Web thickness variations are often minute as the forming and coating processes use closed loop control to limit the thickness variation using scanning on-line mass sensors to infer the web and coating thickness. These minute imperfections in thickness and length across the web width can persist or vary down the web length. Herein it will be shown that these imperfections can result in large variations in stress in the wound roll. The stress variation in the wound roll can dictate the levels of thickness and length variation that are acceptable in the web formation and coating processes.

Winding is an accretive process where thousands of layers of web can be wound onto a core. As layers accrete pressure builds within the roll. Web materials are often orthotropic but the radial modulus of elasticity of webs is known to be state dependent on pressure. This state dependency is the result of a combination of geometric and material nonlinearity factors that are web dependent. The radius of an outer layer in a winding roll varies with thickness non-uniformity and winding conditions. The total web tension is controlled in closed loop with feedback from a web tension sensor that assesses the total tension. The total web tension is parsed across the width of the outer layer of a winding roll depending on the variation in radius of the outer layer. As the outer layer is added to the winding roll some of the web tension is lost due to compression of web layers beneath the outer layer and thus geometric nonlinearity due to large deformations exists. The

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http://dx.doi.org/10.1016/j.cirpj.2014.10.004
1755-5817/

Nomenclature

A_j	cross sectional area of jth sector
B	strain displacement matrix defined in expression (23)
C	elastic compliance matrix
CMD	cross machine (axial) direction
D^*	differential operator matrix defined in expression (22)
E_c	core rigidity modulus
E_r,	web radial modulus of elasticity due expressions (29) and (30)
E_z,	E_θ web axial and tangential moduli of elasticity respectively
$E_{\theta j}$	tangential modulus for jth sector
F_e	element force vector defined in expression (28)
f_j^1, f_j^2	functions defining the inner and outer boundary of jth sector
G_{rz}	shear modulus
h	constant web thickness
h_j, h_{j+1}	web thicknesses at the right and left edge of the jth sector
h_j^{ave}	average web thickness of the jth sector
H^{ave}	average of the average web thicknesses of all sectors
J	Jakobien
K_e	element stiffness matrix defined in expression (27)
K_1, K_2	material constants for radial modulus used in expressions (29) and (30)
L_{ave}	average stress free length of web required for wrapping a layer
L_j	stress free length of web required for wrapping a layer around jth sector
LVC_j	length variation factor for jth sector defined as L_j/L_{ave}
m	number of sectors in CMD
M	elastic material stiffness matrix
MD	machine direction, the direction down the length of the web
n	number of half waves in buckled shape in the y direction
U^*	vector of nodal displacements
u, q	radial and axial displacement fields respectively
r, z	radial and axial coordinates respectively
$\vec{r}$, $\vec{z}$	vectors of nodal displacement for radial and axial directions
r_c	core diameter
r_r	relaxation radius defined in expression (37) and $r_{r1} = r_r$
r_{r2}, r_{r3}	2nd and 3rd iteration of relaxation radius
T_w	nominal web tension, units of stress and $T_{\theta 1} = T_w$
$T_{\theta 2}$, $T_{\theta 3}$	2nd and 3rd iteration of tangential stress at the outer lap
w	constant width for sector
w_j	width for jth sector
ε	strain vector
ν_{rz}, $\nu_{r\theta}$	out-of plane Poisson's ratios
$\nu_{\theta z}$	in-of plane Poisson's ratio
Φ	shape function vector defined in expression (20)
Ψ	shape function matrix defined in expression (26)
η, ξ	natural coordinates
σ	stress vector
σ_0	initial stress vector
$(\sigma_0)^j$	initial stress vector for jth sector, defined in expression (38)
σ_θ, σ_z	stresses in tangential and CMD directions
σ_{rz}	shear stress in the roll

combination of the accretive nature of winding, the material and geometric nonlinearity and the need to distribute web tension in the outer layer due to radius variation leads to inefficiency when attempting to model winding with commercial codes. Herein a novel axisymmetric winding model formulation based on the finite element method is developed which accounts for web imperfections in thickness and length. The development is based on a pre-stress concept in which the nonlinear orthotropic material properties are taken into consideration via layer-wise linearization. The formulation intrinsically accounts for large deformation which has been proven important for soft web materials. The accuracy of the model is demonstrated via core pressure and roll outer radius comparisons to test data on a web with known thickness and length variation. Finally charts of the axisymmetric stress fields will be presented to allow the reader to comprehend the levels of wound roll residual stress variation that can result from actual web thickness variation.

1D winding models

A benchmark for one dimensional winding models was established with the development of Hakiel [1] based on elasticity. This model was the first to combine all of the important influences that govern the behavior of the winding of elastic web materials. Similar to other one dimensional models Hakiel assumed that the spiral geometry of a web wound onto a core could be replaced by concentric rings of web material. This model accounted for stiffness of an elastic core and properly treated the web material properties, including the radial modulus of the wound roll (E_r) which is state dependent on pressure or radial strain. This model was a second order differential equation written in terms of radial stress increments and the first and second derivatives of these stress increments with respect to radius. The radial stress increments were due to the addition of the most recent web layer to the outside of the roll.

$$r^2\frac{d^2\delta\sigma_r}{dr^2} + 3r\frac{d\delta\sigma_r}{dr} - \left(\frac{E_\theta}{E_r} - 1\right)\delta\sigma_r = 0 \qquad (1)$$

One coefficient in this differential equation included the radial modulus (E_r) which was known to be state dependent on the radial stress (σ_r) and hence radius and thus the differential equation could not be solved in closed form. Instead it was solved numerically many times using a finite difference approach, sometimes as many times as there were layers of web in the wound roll. Each time the equation was solved two boundary conditions were required since the equation was second order. One boundary condition resulted from an assumption that the winding tension (T_w) was known from which the radial stress beneath the outer lap could be determined:

$$\delta\sigma_r|_{r=s} = -\frac{T_w|_{r=s}}{s}h \qquad (2)$$

The second boundary condition was based on continuity in deformation at the core. The deformation of the outside of the core