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Procedia CIRP 46 (2016) 151 – 154
www.elsevier.com/locate/procedia

7th HPC 2016 – CIRP Conference on High Performance Cutting

Robust stability of machining operations in case of uncertain frequency response functions

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Abstract

Prediction of machine tool chatter requires a dynamic characterization of the machine-tool-workpiece system by means of frequency response functions (FRFs). Stability lobe diagrams are sensitive to the uncertainties of the measured FRF, which reduces the reliability of their industrial application. In this paper, a frequency-domain method is presented to determine robust stability boundaries with respect to the uncertainties of the FRF. The method is based on an envelope fitting around the measured FRFs combined with some considerations of the single-frequency method. The application of the method is validated on a turning operation characterized by a series of FRF measurements. It is shown that stability analysis using the averaged FRF may considerably overestimate the region of robust stability.

Peer-review under responsibility of the International Scientific Committee of 7th HPC 2016 in the person of the Conference Chair Prof. Matthias Putz

Keywords: Turning; chatter; uncertainty; frf; robustness

1. Introduction

Reliable prediction of machining parameters without producing machine tool chatter is a highly important task for machine tool centers. One of the most accepted explanation for machine tool chatter is the surface regeneration: the vibrations of the tool are copied onto the surface of the workpiece, which modifies the chip thickness and induces variation in the cutting force one revolution later [1], [2]. From dynamic systems point of view, chatter is associated with the loss of stability of the steady-state (chatter-free) machining process followed by a large amplitude self-excited vibration between the tool and the workpiece usually involving intermittent loss of contact. Stability properties of machining processes are depicted by the so-called stability lobe diagrams, which plot the maximum stable depths of cut versus the spindle speed. These diagrams provide a guide to the machinist to select the optimal technological parameters in order to achieve maximum material removal rate without chatter.

Stability lobe diagrams can be calculated using frequency domain techniques, such as the single frequency solution, the multi-frequency solution [3] [4], and the extended multi-frequency solution [5]. These methods apply the measured frequency response functions (FRFs) directly. In contrast, time-domain solutions, such as the semi-discretization method [6], full-discretization method [7], spectral element method [8] or the implicit subspace iteration method [9], just to mention a few, require fitted modal parameters as input. In spite of the available efficient numerical techniques, experimental cutting tests do not always match the predicted stability lobe diagrams [10]. One reason for these differences is the uncertainties of the measured FRFs. For frequency-domain methods, these uncertainties directly affect the generated stability lobe diagrams. For time-domain methods, the uncertainties of the FRF are manifested as uncertainties of the fitted modal parameters, which, again, affect stability lobe diagrams. In this case, the number of modes to be involved in the fitting and the properties of the mechanical model used for the fitting (e.g., proportional vs. non-proportional damping, symmetric vs. non-symmetric FRF matrix) also strongly affect the structure of the stability lobe diagrams [5], [11].

Peer-review under responsibility of the International Scientific Committee of 7th HPC 2016 in the person of the Conference Chair Prof. Matthias Putz
doi:10.1016/j.procir.2016.03.181

A robust method called Edge Theorem combined with Zero Exclusion condition is presented in [12], [13], which can be applied for low number of uncertain parameters. The algorithm, however, leads to intensive numerical computation with many limitations, hence the „robust formulation cannot accommodate more than two varying parameters due to increasing model complexity" [13].

In this paper, a frequency-domain technique is presented for the calculation of the robust stability boundaries in case of uncertain FRFs. The method is based on an envelope fitting around the measured FRF combined with the concept of the single frequency solution, which essentially reduces the computational effort.

2. Mechanical model of turning operations

The dynamical model of an orthogonal cutting operation with multiple vibration modes is shown in Fig. 1. The modes are projected to direction z, which is the direction of the surface regeneration. A multiple-degree-of-freedom model is considered with n number of modes and with the generalized coordinates $\mathbf{z}(t) = (z_1(t), z_2(t) \ldots z_n(t))^{\mathrm{T}}$. The cutting force acting on the tool tip assuming a nonlinear characteristics can be given as

$$F_z(t) = K_z w h^q(t), \tag{1}$$

where K_z is the cutting-force coefficient in direction z, w is the depth of cut, $h(t)$ is the instantaneous chip thickness and q is the cutting-force exponent. The chip thickness is affected by the current tool position and the previous position of the tool one revolution before. The regenerative time delay is $\tau = 60/\Omega$, where Ω is the workpiece revolution given in rpm. The instantaneous chip thickness then can be calculated as

$$h(t) = v_{\mathrm{f}}\tau + z_1(t-\tau) - z_1(t), \tag{2}$$

where v_{f} is the feed velocity and $z_1(t)$ indicates the position of the tool tip in direction z. The linearized forcing vector $\mathbf{f}(t)$ is

$$\mathbf{f}(t) = \begin{pmatrix} K_z w q (v_{\mathrm{f}}\tau)^{q-1}\big(z_1(t-\tau) - z_1(t)\big) \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \tag{3}$$

3. The single-frequency method

In order to obtain a frequency-domain estimation of the robust stability boundaries, the main steps of the single-frequency method is summarized for the stability analysis of the systems with fixed FRF [14]. The definition of the FRF matrix $\mathbf{H}(\omega)$ gives

$$\mathbf{H}(\omega)\mathbf{F}(\omega) = \mathbf{Z}(\omega), \tag{4}$$

where $\mathbf{F}(\omega)$ and $\mathbf{Z}(\omega)$ are the Fourier transforms of the forcing vector $\mathbf{f}(t)$ and displacement vector $\mathbf{z}(t)$, respectively. The Fourier transform of the linearized parametric forcing $\mathbf{f}(t)$ is

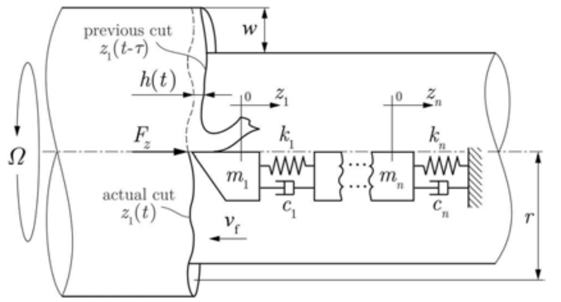

Fig. 1. Surface regeneration in a turning process with multiple vibration modes. Parameters: w-depth of cut, $h(t)$-chip thickness, v_{f}-feed rate, Ω-spindle speed, F_z-cutting force in direction z, z_i-modal coordinates, c_i-modal damping, k_i-modal stiffness, m_i-modal mass, r-workpiece radius.

given as

$$\mathbf{F}(\omega) = \boldsymbol{\kappa}(\mathrm{e}^{-\mathrm{i}\omega\tau} - 1)\mathbf{Z}(\omega), \tag{5}$$

where

$$\boldsymbol{\kappa} = \begin{pmatrix} \kappa & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \quad \text{and} \quad \kappa = K_z w q (v_{\mathrm{f}}\tau)^{q-1} \tag{6}$$

is the specific cutting-force coefficient. Substitution of Eq. (4) into Eq. (5), and simplification yield

$$\big(\mathbf{I} - (\mathrm{e}^{-\mathrm{i}\omega\tau} - 1)\boldsymbol{\kappa}\mathbf{H}(\omega)\big)\mathbf{F}(\omega) = \mathbf{0}, \tag{7}$$

where $\mathbf{I}$ is the identity matrix. The existence of the nontrivial solution implies

$$\det\big(\mathbf{I} - (\mathrm{e}^{-\mathrm{i}\omega\tau} - 1)\boldsymbol{\kappa}\mathbf{H}(\omega)\big) = 0, \tag{8}$$

which, considering the structure of $\boldsymbol{\kappa}$, can be expressed as

$$D(\omega) = 1 - (\mathrm{e}^{-\mathrm{i}\omega\tau} - 1)\,\kappa H(\omega) = 0. \tag{9}$$

Here, $H(\omega) := H_{11}(\omega)$ is the measured tip-to-tip FRF. If the inverse FRF is written as $H(\omega) = \Lambda_{\mathrm{Re}}(\omega) + \mathrm{i}\Lambda_{\mathrm{Im}}(\omega)$, then the analytic solution for the stability lobe diagrams, where Eq. (9) is satisfied, can be given as [4]

$$\Omega = \frac{30\omega}{\arctan\left(\dfrac{\Lambda_{\mathrm{Re}}(\omega)}{\Lambda_{\mathrm{Im}}(\omega)}\right) + j\pi} \quad \text{and} \quad \kappa = -\frac{\Lambda_{\mathrm{Re}}(\omega)^2 + \Lambda_{\mathrm{Im}}(\omega)^2}{2\Lambda_{\mathrm{Re}}(\omega)^2}, \tag{10}$$

where $j = 1, 2, \ldots, \infty$ and $\omega \in [0, \infty)$. These parametric curves gives the stability lobes in the parameter plane (Ω, κ).

4. Robust stability analysis

The FRFs obtained from measurements are always loaded by noise and uncertainties, which can be represented as an uncertain envelope around the averaged FRF. In this paper it is