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2-D elasticity solutions of two-layer composite beams with an arbitrarily shaped interface

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ABSTRACT

Based on the two-dimensional (2-D) elasticity equations, an analytical model for analyzing simply supported two-layer composite beams with an arbitrarily shaped interface under transverse load is proposed. By using Fourier series expansion, the solution of stresses and displacements for the beam is obtained. In numerical examples, the convergence of the solution is demonstrated and the solution shows a good agreement with the solution obtained from the finite element software. Numerical examples also show that geometric parameters of the interface have an effect on the distributions of stresses and displacements of the beam. By choosing suitable geometric parameters of the interface and without changing the amount of materials, some variables about stresses and displacements can have significant decreases in comparison with those of traditional composite beams with horizontal interfaces.

1. Introduction

The excellent mechanical properties of composite structures such as high specific stiffness and high specific strength make them suitable for widespread applications in structural engineering. The most common applications are steel-concrete beams and wood-concrete beams. In recent years, sandwich structures [1–3], laminated glass structures [4–7] and many more are received extensive attentions. The mechanical performance of the composite structures depends not only on the material properties of each layer, but also on the stiffness of interfacial connection between the adjacent layers. Commonly used composite structures have uniform thickness for each layer, while composite structures with non-uniform layered thickness are sometimes used to satisfy special functional requirements, especially are used in repair of structures with damages such as the new-to-old concrete structures [8], i.e. the old concrete structure strengthened by a new concrete layer.

1.1. Summary of the state of the art

A review of the literature indicates that a lot of analytical models have been proposed for analyzing composite structures. To cite a few, the moving least square differential quadrature method was applied to bending and buckling of symmetric and antisymmetric laminates by Huang et al. [9,10]. According to global higher-order deformation theories, Matsunaga [11] analyzed interlaminar stress of laminated composite beams. Girhammar and Gopu [12] developed the differential equations for analyzing beam-columns with interlayer slip. By using the Timoshenko beam theory, Schnabl et al. [13] analyzed two-layer

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beams with interlayer slip. Based on exact elasticity equations, Xu and Wu [14] developed a plane stress model, without assumptions of the transverse shear deformation, for two-layer beams with interlayer slip. Foraboschi [15] proposed an analytical model to predict stress development and ultimate strength of laminated glass beams. Foraboschi [16] developed a nonlinear model for analyzing two-layer beam with interlayer slip. Galuppi and Royer-Carfagni [17] proposed an analytical model for analyzing time-dependent behavior of laminated beams with a viscoelastic interlayer. Based on the Kirchhoff-Love plate theory, Foraboschi [18,19] proposed an analytical model for analyzing a three-layer plate made of two identical (stiff) outer layers and a more compliant inner interlayer. Asik [20] studied nonlinear behavior of laminated glass plates undergo large deflections.

The composite structures in these analytical studies are limited to have plane interfaces. In essence, study on composite structures with non-planar interfaces is necessary. For example, new-to-old concrete structures usually have non-planar interface between new and old concrete layers [8]. Based on the finite element method, Shi and Shimoda [21] proposed interface shape optimization of designing functionally graded (FG) sandwich structures. They found that the compliance of FG sandwich structures can be reduced after optimizing and the optimized interface is non-planar. To the best of the author's knowledge, no analytical model of composite structures with non-planar interfaces has been reported yet. To fill this gap, in this paper, an analytical model of a simply supported two-layer composite beam with an arbitrarily shaped interface under transverse load is proposed and analytical solutions are obtained.

There are a series of theories for analyzing composite structures. The classical Euler–Bernoulli beam theory is well known as elementary theory of bending. This theory neglects the effect of transverse shear deformation on bending solution, since it is based on the assumption that plane cross-sections normal to the neutral axis remain plane and are perpendicular to neutral axis during bending. This implies the infinite shear stiffness. Actually, no material possesses such a property. This beam theory is very successful for analyzing slender homogeneous beams, whereas it may be not sufficiently accurate in circumstances where shear deformation can be significant, as in thick and short beams, where the span-to-depth ratio is small and the flexural-to-shear rigidity ratio is large. Timoshenko was the first to investigate thick beams taking account of transverse shear deformation. This theory is referred to the Timoshenko beam theory in the literature. In Timoshenko's theory, plane cross sections remain plane but not necessarily perpendicular to neutral axis during bending, meanwhile the distribution of the transverse shear strain along the beam thickness is assumed to be constant and is expressed by shear correction factor. The accuracy of the solution based on the Timoshenko beam theory largely depends on predicting suitable shear correction factor.

To improve the accuracy of the transverse shear deformation prediction, higher-order theories taking account of transverse shear deformation have been proposed [22–25]. Most of these theories are third-order theories in which the transverse displacement and axial displacement are, respectively, assumed to be quadratic expression and cubic expression at most. In the higher-order theories, the transverse and axial displacements are assumed to be C^1 to C^∞ continuous in the consistent conditions according to the order of the theories.

Some analytical solutions based on exact elasticity theory have been developed [26–28]. Since the elasticity theory renounce any assumptions of the transverse shear deformation, the solutions based on elasticity theory generally more accurate than those based on the Euler–Bernoulli theory, Timoshenko's theory and higher-order theory.

In this paper, an analytical model based on exact elasticity theory is proposed for analyzing the two-layer composite beam with an arbitrarily shaped interface. By using Fourier series expansion, the solution of stresses and displacements for the beam is obtained. Numerical examples indicate convergence and accuracy of the solutions, meanwhile the geometric parameters of the interface are investigated.

2. Analytical model

Consider a 2-D simply supported two-layer beam with an arbitrary interface subjected to transverse loads, as shown in Fig. 1. The beam is composed of two non-uniform layers with different materials. The length and total thickness of the beam are l and h, respectively. The thicknesses of the layer 2 at the left end and the right end are denoted by h_1 and h_2, respectively. The geometry shape of the interface between the two layers is denoted by a continuous function $f(x)$. The elasticity modulus and Poisson's ratio are denoted by E_i and μ_i, respectively. The distributed transverse loads acted on the top and bottom surfaces are denoted by $q_1(x)$ and $q_2(x)$, respectively. The symbol i denotes that the quantities belong to layer i ($i = 1, 2$).

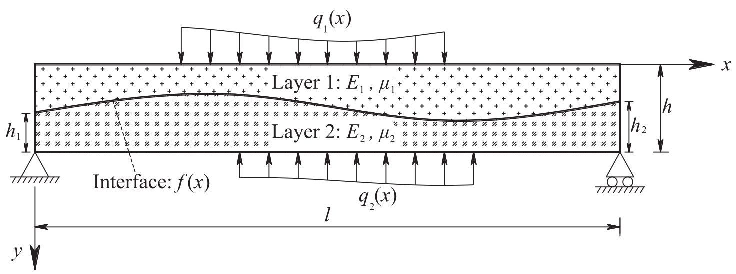

Fig. 1. Simply supported two-layer beam with an arbitrarily shaped interface.