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New results on the coordination of transportation and batching scheduling

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ABSTRACT

We study a planning problem to coordinate production and transportation scheduling, where a set of jobs needs to be transported from a holding area to a single batch machine for further processing. A number of results for this combined transportation-and-scheduling environment have recently been published. They look into the complexity status of the minimization of the sum of total processing time and processing cost, and of the sum of makespan and processing cost, for a fixed number of transporters. In this paper, we add to these results in that (1) we show that the earlier complexity results are still valid when the processing cost is removed from the objective, thus reducing to more “classic” scheduling objectives; (2) we assess the complexity status of the relevant problem variants with free number of transporters; and (3) we prove that the weighted-completion-time objective leads to an intractable problem even with a single transporter, contrary to the unweighted case.

1. Introduction

Tang and Gong [1] address a planning problem that coordinates transportation and batch processing in the iron and steel industry. A set $N = \{1, 2, \ldots, n\}$ of jobs is initially located at a holding area, and each of the jobs needs to be transported by one of m available vehicles before it can be processed by a single batch machine. Each vehicle can transport one job at a time. We let t_j denote the transportation time for job $j \in N$ from the holding area to the machine, and t the (empty) vehicle return time from the machine back to the holding area (to pick up a new job). All vehicles are assumed to be located in the holding area at the start of the planning horizon. In the production stage, up to c jobs can be processed as one batch on the batching machine; the processing time of (all jobs of) each batch is equal to p, which is a constant. Following Tang and Gong [1], the resulting optimization problem is denoted as TBS, short for *transportation and batching scheduling problem*. A formal definition is as follows:

TBS. Input: job set $N = \{1, 2 \ldots, n\}$, number of vehicles $m \in \mathbb{N}$, transportation time $t_j \in \mathbb{Q}$ for each $j \in N$, vehicle return time $t \in \mathbb{Q}$, batch machine capacity $c \in \mathbb{N}$, batch processing time $p \in \mathbb{Q}$, and objective function $F(y_1, y_2, \ldots, y_n, y_{n+1})$, with $y_j \in \mathbb{Q}$ $(j = 1, \ldots, n)$ and $y_{n+1} \in \mathbb{N}$. Goal: find a vehicle assignment $N \to \{1, \ldots, m\}$, a transportation sequence for all jobs assigned to the same vehicle, and a partition $B_1, B_2, \ldots, B_b$ of N such that objective function $F(C_1, C_2, \ldots, C_n, b)$ is minimized, where b is the number of batches processed on the batching machine, set B_l contains the jobs processed in the lth batch, $B_l \le c$ for $l = 1, \ldots, b$, and C_j is the completion time of job j on the batching machine $(j \in N)$ when the schedule starts at time 0.

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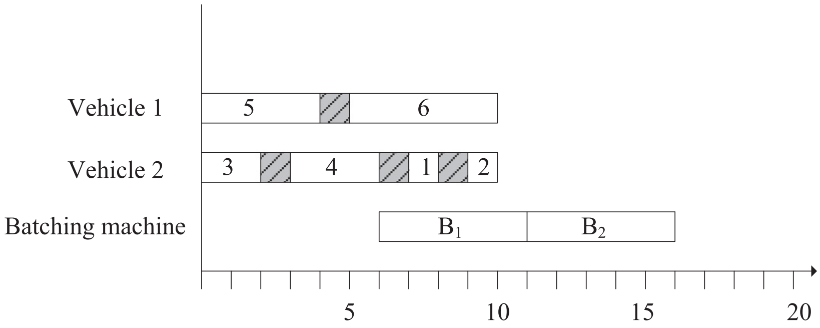
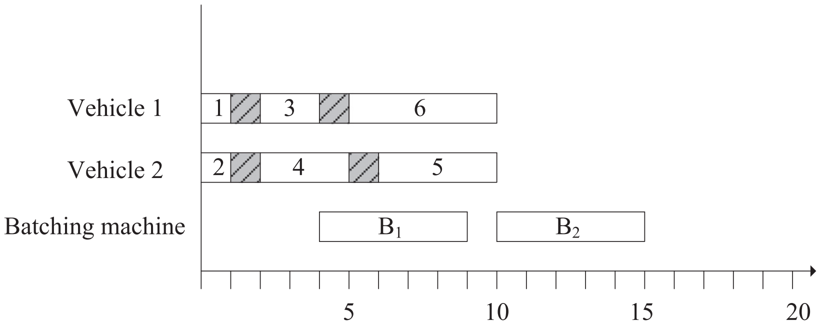


(a) a feasible schedule



(b) an optimal schedule

Fig. 1. Two schedules for the example instance.

Note that the transportation times t_j ($j \in N$) are the only feature in which jobs differ from each other. Consequently, with identical transportation times the instance size would no longer be proportional to n. We illustrate this problem definition with the following example instance: we have $m = 2$ vehicles and $n = 6$ jobs with transportation times 1, 1, 2, 3, 4 and 5 (which are the values of t_1 to t_6, respectively). The batching machine can process at most three jobs at a time, so $c = 3$, and the duration of one batch run is $p = 5$. The empty return time from the batching machine back to the holding area equals $t = 1$. Fig. 1(a) depicts a feasible schedule with makespan 16, which uses $b = 2$ batches: the first batch B_1 consists of jobs 3, 4 and 5 and B_2 contains job set {1, 2, 6}. The grey hatched blocks indicate that the vehicle returns to the holding area. A better schedule for makespan minimization is given in Fig. 1(b), with makespan 15. It can be seen that this is a schedule with the lowest possible makespan because at least one job will always arrive at the batching machine at time 10 at the earliest. Note that with this job set, it might still be interesting to use more than two batches with other objective functions, but not for makespan.

Below, we will discuss the complexity status of TBS for subproblems corresponding with specific choices for the objective function F. Tang et al. [2] provide an elaborate description of the practical relevance of this type of scheduling problems in the context of ingot processing in the steel industry. Steel ingots are created by pouring molten steel into molds placed on a vehicle; the steel then solidifies. The molds are subsequently removed (“stripped”) from the ingots and the ingots are transported to a soaking pit by the vehicles. The transportation time of the ingots in each trip (representing the transportation phase) includes solidifying, stripping and actual travelling, and Tang et al. [2] describe how different jobs may have different transportation times because the time required for solidifying and stripping depends on the steel grade as well as on other attributes of the ingots. In the soaking pit, multiple ingots are reheated simultaneously to be prepared for rolling (this reheating is the batching phase).

From a practical perspective, Tang et al. [2] observe that (again in steel ingot processing) the fuel cost is directly proportional to the number of batches, which explains the desire to balance a time-related and a cost-related objective. Tang and Gong [1] investigate TBS when $F(C_1, \ldots, C_n, b) = \sum_{j \in N} C_j + \alpha(b)$, the sum of total completion times and a processing cost $\alpha(b)$ that depends on the number b of batches. They prove that the problem is NP-hard even if $m = 2$ and present a pseudo-polynomial-time algorithm and FPTAS for any fixed m. Therefore, TBS to minimize the sum of total completion times and processing cost is NP-hard in the ordinary sense for any fixed $m \geq 2$. For $m = 1$, the problem turns out to be polynomially solvable. Zhu [3] shows that TBS with $F(C_1, \ldots, C_n, b) = C_{\max} + \alpha(b)$ is NP-hard even if $m = 2$, where $C_{\max} = \max\{C_1, \ldots, C_n\}$, the maximum completion time or makespan . Using a method similar to [1], he provides a pseudo-polynomial-time algorithm and FPTAS for any fixed m, thus again concluding NP-hardness in the ordinary sense; he also describes a polynomial-time algorithm for TBS with $F = C_{\max} + \alpha(b)$ when