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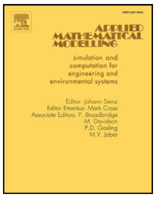

An analytical solution for a transient temperature field during laser heating a finite slab

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ABSTRACT

This paper presents a general solution for the transient temperature field in a finite slab, when heated by a moving laser heat source. The analytical solution was solved by integral transform method. The eigenvalues were obtained by the method of separation of variables. In the presented model, finite region and general convective boundary conditions were assumed. Meanwhile, experiment and FEM simulation were performed as a verification under a specific case. For the experiment, a continuous wave YAG laser system was used to scan an aluminum sheet, and temperature histories at various points were logged for comparison. For the FEM simulation, a 3D transient heat transfer model was adopted with a subroutine to implement the moving laser beam. The results showed that the analytical solution was consistent with both the experimental data and FEM simulation.

1. Introduction

Lasers are widely adopted in material processing because of their highly concentrated energy distribution. There are many applications of lasers in sheet metal forming such as laser forming and laser assisted pre-stress forming. Laser forming is a flexible technique which forms sheet metal by means of laser induced non-uniform temperature field and thermal stress [1]. During the forming process, deformation is induced in a controllable manner by planning the laser scanning path in advance. It is characterized as a non-contact and non-wearing treatment. On the other hand, in laser-assisted pre-stress forming, forming is achieved by radiating strain concentrated areas with lasers, where the strain is induced by a pre-loaded stress [2]. Due to the softening effects in the heated region, material yield strength is locally reduced and the part is formed effectively. In all these forming methods, there are many factors governing the final formed shape, and even the forming mechanisms, such as laser parameters, part geometry dimensions, and boundary constraints. It was reported to be viable to predict forming ratio or bending angle in those two major laser forming techniques directly by adopting empirical modeling, without solving the temperature field. For example, Cheng and Yao [3] developed a method for laser forming of a class of shapes with genetic algorithm. Springback ratio can be predicted and controlled by modeling with neural network and large set of experimental data [4]. Pulsed laser bending of sheet metal can be modelled and optimized with neural networks and neuro-fuzzy system [5]. However, since deformation is driven by thermal field, temperature field would give a better understanding in the forming mechanisms. On the other hand, temperature field is of great importance in procedure planning, which is a dominant key for desired or undesired material transition. For instance, there is a maximal permissible temperature value to ensure that the workpiece is not compromised during

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forming process [2]. For such purposes, efforts have been made to solve the temperature field via various approaches such as numerical simulations, artificial intelligence techniques and analytical methods.

Numerical simulations, finite element method (FEM) or finite difference method (FDM) in most of the cases, are effective to analyze the temperature profile with given specific laser processing parameters. Ji and Wu [6] established a simplified mathematical model for the laser heating of sheet metal, analyzed the heating process with a FEM program, and compared with a FDM model. Chen et al. [7] proposed a numerical model, which combined equations of FEM with those of FDM, for the simulation of the temperature filed. Jamil et al. [8] investigated influences of different laser beam geometries on temperature profiles with FEM simulation. Yilbas and Akhtar [9] studied temperature history under multiple laser scans by introducing a volume flux in ABAQUS. With proper simplifications and assumptions, FEM simulation with a fine mesh can approximate the real-world situation. However, FEM approach is commonly much time-consuming, given that the computing cost increases with a finer mesh [10].

Recently, researchers have employed various artificial intelligence techniques for the prediction and optimization of various laser advanced machining process. Artificial Neural Network (ANN) has been widely used for modeling because of its nonlinear, adaptive, and learning properties. Ismail et al. [11] used an ANN model to establish an intelligent algorithm to build a simplified relationship between laser parameters and weld bead geometry for laser microwelding of thin steel sheet. Mishra and Yadava [12,13] proposed an ANN model, which was trained with verified FEM model and simulated data, to predict the hole taper and the material removal rates as well as the extent of HAZ during laser beam percussion drilling of both thin aluminum sheet and nickel-based super-alloy sheet. ANN is widely established in the artificial intelligence research where the output variables behavior non-linearly with the input variables. To approximate such non-linear relationship, a large set of experimental or simulation data is commonly required, especially in the area of advanced laser machining.

In sum, both ANN and FEM can be utilized to predict the real-world situation. On the other hand, analytical method provides a reasonably simplified model of abstraction by directly solving the governing differential equation for heat transfer. It is worthy to mention that ANN model, accompanied with an analytical model of a one dimensional heat transfer equation, does predict the achieved hardness in laser hardening of K340 steel [14].

To model the temperature field during laser forming analytically, Geiger and Vollertsen proposed a two-layer model, which presented a temperature discontinuity in the middle [1]. This model could fit the experimental data only on the very specific conditions. Vollertsen and Rodle proposed another model taking account of temperature distribution along thickness [15]. Lambiase [16] also established another two-layer model whereas the heated layer thickness depended on the effective temperature distribution along the sheet thickness. However, a stationary source was assumed and distributions along the length and the width direction were neglected. In some cases, one-dimensional model is suitable for modeling the temperature distribution along depth direction. Nath et al. [17] used such a model to investigate the temperature variations during heating and cooling cycles in laser surface hardening. Lambiase and Di Ilio [18] predicted the bending angle during laser forming by dividing and averaging accordingly the temperature field along depth direction into two layers, based on the 1-D thermal modeling. Cheng and Lin [19] proposed an analytical model with a moving Gaussian distribution source by solving the heat transfer equation with superposition and mirror-image method. Yet the solution assumed a semi-infinite slab with adiabatic boundary condition on both surfaces. Conde et al. [20] proposed two models depending on the modeling of laser source using the Green function method. However, the boundary conditions were also considered to be adiabatic. Majumdar and Xia [21] proposed a Green's function model for a two dimensional situation. Because of the absence of width direction, the effect of width on the temperature distribution cannot be calculated.

As a matter of fact, for many cases, adiabatic boundary conditions cannot be assumed to predict temperature field with enough precision. For example, Lambiase et al. proposed a passive water cooling method during laser forming process, which can be beneficial in bending angle and in cooling time [22]. In such case, intensive convective boundary condition should be considered. As another example, during laser assisted pre-stress forming of aluminum sheet, the maximal temperature is suggested to be no higher than 300°C; convective boundary significantly affects the temperature field [23].

In this paper, an analytical model for a transient temperature field during laser heating a finite slab was proposed by solving directly the general 3D heat transfer equation. Such solution was obtained by integral transform method, accompanied with separation of variables for solving the corresponding homogenous equation. To verify the model, a case that an aluminum workpiece was heated by laser line scanning was chosen. In addition, both experiment and FEM simulation were performed. Experiment was carried out by heating an aluminum sheet with continuous wave YAG laser, and logging the temperature history at interested test points. FEM simulation was realized with ABAQUS software. Verification indicated that the presented model was feasible under the given processing parameters.

2. Mathematical modeling

The problem of heating a finite slab with a laser beam of a specific energy distribution can be modelled as a three dimensional heat transfer equation, in a region of 3D Euclidean space, as shown in Fig. 1. The top surface is radiated with a moving laser beam with a specific power distribution. The length, width and thickness are denoted by L, W and H respectively. Laser beam radius is denoted by r_b. And P1, P2, and P3 are points where temperature histories are measured for model verification. The governing heat transfer equation and the boundary and initial condition can be formulated with:

$$\frac{1}{D}\frac{\partial T(x,y,z,t)}{\partial t} - \nabla^2 T(x,y,z,t) = \frac{1}{k} g(x,y,z,t), \quad \text{for } (x,y,z) \in B, \tag{1}$$