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Finite-time stability of impulsive fractional-order systems with time-delay

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ABSTRACT

In this paper, a class of impulsive fractional-order systems with time-delay is investigated. Sufficient conditions for the finite-time stability(FTS) of impulsive fractional-order systems with time-delay are established by the generalized Gronwall's inequality. It shows that the FTS of such systems depends on the frequency and amplitude of the impulses. Examples are given to illustrate the results.

1. Introduction

In the past two decades, fractional-order systems have been intensively studied due to its wide applications to various fields, such as viscoelastic systems, dielectric polarization, electromagnetic waves, heat conduction, robotics, biological systems, finance, and so on, see, for example, [1–3].

As we know, practical applications heavily depend on the dynamical behaviors, especially the stability, of models. So the stability of fractional differential equations(FDEs) has become one of the most active areas of research, and has attracted increasing interests from many scientists and engineers, see, for example, [4]. for a survey of the stability of FDEs.

Impulsive dynamical systems, which can be viewed as a subclass of hybrid systems, have not only played an important role in modeling physical phenomena subject to abrupt changes, but also from the control point of view provided a powerful tool for stabilization and synchronization of chaotic systems [5]. For the theory of impulsive dynamical systems and its applications, refer to [6,7]. and references therein.

In addition, impulsive fractional differential equations have also attracted considerable interests amongst researchers. It's worth mentioning that Fečkan et al. [8] introduced a formula for solutions of the Cauchy problem of impulsive fractional differential equations with Caputo derivative and gave a counterexample to show that the previous results were incorrect. The related existence, uniqueness and data dependence results were presented in [9]. A pioneering work on the Hyers–Ulam–Russia stability for nonlinear impulsive fractional differential equations have been reported by Wang et al. [10]. In [11], some necessary

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and sufficient conditions of controllability and observability for the impulsive fractional linear time-invariant system has been given.

On the other hand, different from classical stability, FTS deals with systems whose operation is limited to a fixed finite interval of time. From practical considerations, FTS seems to be more appropriate for systems whose variables must lie within specific bounds. In [12,13], the FTS of fractional-order systems with time delay was considered.

Until now, there are many valuable results for FTS of integer order impulsive systems [14–17]. Amato et al. [14] dealt with the FTS problem for continuous time linear time-varying systems with finite jumps. In [15], finite-time stabilization problems for continuous-time linear time-varying systems with impulsive control were investigated. In [16], sufficient conditions for linear singular impulsive systems to be finite-time stable were proposed in terms of a set of coupled matrix inequalities. In [17], sufficient conditions for FTS of state-dependence impulsive linear systems were provided. However, to the best of our knowledge, there are few results about the FTS of impulsive fractional-order systems.

The rest of this paper is organized as follows. In Section 2, some notations and definitions of fractional calculus are given. Some preliminary results are also presented. In Section 3, the sufficient conditions for the FTS of a class of impulsive fractional differential equations with time delay are derived. In Section 4, examples are given to demonstrate the effectiveness of the main results.

2. Preliminaries

Generally speaking, there are three commonly used definitions of fractional derivative definitions, i.e., Grunwald–Letnikov fractional derivative, Riemann–Liouville fractional derivative, and Caputo's fractional derivative [3]. The last one is frequently adopted by applied scientists, since it is more convenient in the setting of the initial conditions.

The Caputo's fractional derivative of order $q \in (0, 1)$ with the lower limit zero for a function can be defined by

$$ {}^{c}D^{q}_{0,t} f(t) := D^{q}_{0} f(t) = J^{1-q}_{0} f'(t), $$

where J^{β}_{0} is the βth Riemann–Liouville integral operator, which is expressed as

$$ J^{\beta}_{0} f(t) = \frac{1}{\Gamma(q)} \int_{0}^{t} (t-s)^{\beta-1} f(s) ds, \quad \beta > 0. $$

Here, $\Gamma(.)$ is the well-known Euler's gamma function.

Consider the following impulsive fractional-order system with time-delay:

$$ \begin{cases} D^{q}_{0} x(t) = A(t) f(x) + B(t) g(x(t-\tau)) + h(t), & t \in J/\{t_1, t_2, \ldots, t_m\} \\ x(t) = \varphi(t), & t \in [-\tau, 0] \\ \Delta x(t_k) := x(t_k) - x(t_k^-) = I_k(x(t_k^-)), & k = 1, 2, \ldots, m \end{cases}, \tag{1} $$

where $T \in R, J \equiv [0, T], 0 < t_1 < t_2 < \cdots < t_m < T. A(t) = (a_{ij}(t))_{l\times l} (l \in N^+), B(t) = (b_{ij}(t))_{l\times l}, a_{ij}(t), b_{ij}(t) \in C[0, T], \varphi(t)$ and $h(t)$ are both continuous functions. I_k: $R^l \mapsto R^l$ are continuous for $k = 1, \ldots, m$, $x(t_k^-) = lim_{\varepsilon\to 0^-} x(t_k + \varepsilon)$. $f$and g are Lipschitz continuous, that is, there exist positive constants l_f and l_g such that:

$$ ||f(u) - f(v)|| \le l_f ||u - v||, \quad ||g(u) - g(v)| \le l_g ||u - v||, \quad \forall u, v \in R^l, \tag{2} $$

τ is a positive constant, $f(0) = 0$, $g(0) = 0$.

Definition 1. System (1) is finite-time stable w.r.t $\{\delta, \varepsilon, J\}, \delta < \varepsilon$, if and only if

$$ ||\varphi||_c < \delta \Longrightarrow ||x(t)|| < \varepsilon, \quad \forall t \in J $$

where $||\varphi||_c = sup_{-\tau \le t \le 0} ||\varphi||$, $||.||$ denotes the usual vector norm of R^l.

First, we present some preliminary results. Let $PC^1([-\tau, T], R^l)$ denotes the Banach space of all piecewise right continuous derivative functions from $[-\tau, T]$ into R^l which have left continuous derivative on $[0, T]$ with the norm $||x||_\infty = sup_{t\in[-\tau,T]} ||x(t)||$. Then we have