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Fractional description of time-dependent mechanical property evolution in materials with strain softening behavior

Ruifan Meng, Deshun Yin *, Chao Zhou, Hao Wu

College of Mechanics and Materials, Hohai University, No. 1 Xikang Road, Nanjing, Jiangsu 210098, PR China

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ABSTRACT

On the basis of using the variable fractional order to characterize the time-dependent mechanical property, a variable order fractional viscoelastic model is presented to represent the time-dependent evolution of mechanical property including strain softening behaviors. The developed model is applied to analyze the constant strain rate tension and compression tests in ductile metals and soils. It is shown that the model can reasonably characterize the hardening and softening stages of constant strain rate tests and describe the evolution of mechanical property. The time-dependent mechanical property evolution can be split into three parts: in the first stage, the mechanical property is almost invariable, the second phase is a mutation stage, and in the third stage it changes slowly and linearly with strain. The simulated results reveal that the evolution from strain hardening to strain softening is a continuous linear change of mechanical property. Furthermore, it is known that the order obtained through fitting some test data can be greater than 1, which corresponds to strain softening response, while for a long time the order was always assumed to be between 0 and 1.

1. Introduction

Strain softening behavior is a phenomenon where the stress decreases with further deformation after the stress reaches its peak value and has been found in many materials [1]. Strain softening always follows strain hardening, in which stress is increasing with strain. Thus, strain softening is not a separate mechanical phenomenon. Strain-hardening behaviors of materials have been studied extensively [2], but strain-softening behaviors are less understood. This strain softening characteristic is of great technological importance, as the development of high-effective and low-cost process method with a certain strain rate can be promoted, such as high-speed rolling technology.

It is known that elastic and viscous (or intermediate, viscoelastic) behavior is relevant at short times (transient behavior) [3]. For example, ductile metal will experience elasticity, plasticity and even viscosity in turn under a constant strain rate loading. In other words, the mechanical property is changing during the deformation or loading. As mentioned above, strain softening behavior occurs after strain hardening. Thus, strain softening behavior is actually a result of mechanical property evolution. Since the 1970s great efforts have been undertaken to integrate strain-softening into macroscopic ('phenomeno logical') constitutive equations in the framework of elastic–plasticity, see, e.g. Ref. [4] and references therein. Such material models frequently introduce a negative tangent stiffness and can simulate related phenomena. Nevertheless, the problem of

* Corresponding author. Tel.: +86 13675176567.
E-mail address: yindeshun@hhu.edu.cn (D. Yin).

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mechanical property evolution is not involved in those models. Moreover, it was known that strain softening behavior occurs along with plastic deformation, and different strain rates lead to the appearance of different strain softening characteristics. However, the rate dependence is not considered by most elastoplastic models [5–7]. Some models [8] take into account the influence of rate, but the shape of these models is relatively complex.

Fractional calculus has been considered as one of the best mathematical tools to model physical responses and is particularly suitable for building the time-dependent constitutive model [9]. The use of the fractional calculus is motivated in large part by the fact that fewer parameters are required to achieve accurate approximation of experimental data. It is well known that the ideal solid obeys the Hooke's law, $\sigma(t) \sim \varepsilon(t)$, and a Newtonian fluid satisfies the Newton's law of viscosity, $\sigma(t) \sim \mathrm{d}^1\varepsilon(t)/\mathrm{d}t^1$, where σ and ε are the stress and strain. So it is not difficult to imagine that the 'intermediate' material, which is intermediate between ideal solid and Newtonian fluid should follow

$$\sigma(t) = E\theta^{\alpha}\frac{\mathrm{d}^{\alpha}\varepsilon(t)}{\mathrm{d}t^{\alpha}}, \tag{1}$$

where E and θ are material constants and $0 \leqslant \alpha \leqslant 1$. So if we regard the mechanical properties as a "spectrum", both ends of which are the "pure" elasticity, $\alpha = 0$, and the "pure" viscosity, $\alpha = 1$, the order of Eq. (1) can denote the location of a specific mechanical feature on the "spectrum", which can help us distinguish the mechanical property quantitatively. However we find that some mechanical behaviors still cannot be simulated by Eq. (1). The primary reason is that the constant fractional order in Eq. (1) implicates the invariability of mechanical property while in the real world it is changing during the mechanical process.

A further generalization of the concept of fractional order calculus that is applicable to more complex mechanical property of material is that of a calculus of varying order. Up to now, a number of variable-order fractional calculus definitions have been proposed [10–12] and some of them have been applied into practical fields such as anomalous diffusion [13–15], viscoelasticity [16–18], multifractional Gaussian noises [19], processing of geographical data [20], FIR filters [21], etc. Following the idea of using the fractional order to reflect mechanical property, the variable-order operator should be able to depict its evolution during loading. In most existing viscoelastic applications of fractional order calculus, the order is assumed to be between 0 and 1, which means that the mechanical property of material can only be variable between elasticity and viscosity. It is still unclear whether the order can exceed 1. Moreover, we do not know if the variable order fractional calculus can be employed to characterize the strain softening.

A better understanding of strain-softening behavior would improve the level of industrial processes. Thus, in this paper, we attempt to describe the evolution of mechanical property for the mechanical response with strain softening under the condition of uniaxial tension of constant strain rate utilizing the variable order calculus, and then discuss the relationship between the order and the strain softening.

2. Fractional order calculus and variable order fractional calculus

2.1. Fractional order calculus

The fractional order derivatives have been widely used in many areas, for example, polymers, electromagnetic, viscoelastic mechanics, non-Newtonian fluid, geomaterials and so on. In order to use the fractional order in engineering, the Caputo definition was presented, which is the most frequently used definition in engineering now. Its definition is as follows [22]:

$${}_aD_t^{\beta}f(t) = \int_a^t \frac{(t-\tau)^{n-\beta-1}}{\Gamma(n-\beta)} f^{(n)}(\tau)\mathrm{d}\tau. \tag{2}$$

Definition can be also written as:

$${}_aD_t^{\beta}f(t) = \frac{1}{\Gamma(n-\beta)}\int_a^t (\sigma-a)^{n-\beta-1}f^{(n)}(t-\sigma+a)\mathrm{d}\sigma. \tag{3}$$

2.2. Variable fractional order calculus

From the mathematical viewpoint, a fractional derivative may be written as [18]

$${}_0D_t^{\alpha}f(t) = g(t,\alpha). \tag{4}$$

It also may be rewritten as

$${}_0D_t^{\alpha}f(t) = \sum_{k=1}^{n}[g(t_k,\alpha) - g(t_{k-1},\alpha)] + g(0,\alpha), \quad (0 = t_0 < t_1 < t_2 \cdots < t_n = t), \tag{5}$$

where α is constant. When we assume that the order in Eq. (5) is a piece-wise constant manner, shown in Table 1, Eq. (5) can also be written as