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Camouflage devices with simplified material parameters based on conformal transformation acoustics ☆

Chunyu Ren [a,b], Zhihai Xiang [a,*]

[a] AML, Department of Engineering Mechanics, Tsinghua University, Beijing, China
[b] Department of Naval Architecture and Ocean Engineering, Naval University of Engineering, Wuhan, China

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ABSTRACT

Owing to the recent fascinating advances in transformation acoustics, it is possible to design the distribution of refractive index of attractive devices that can manipulate acoustic waves in almost any manner. Furthermore, if the transformation is conformal, the resultant devices can be made with ordinary isotropic materials instead of exotic anisotropic metamaterials. In this paper, we use conformal transformation acoustics to design camouflage devices with layered homogeneous structures, which can acoustically generate illusions of objects. We demonstrate two devices: one is called shifter that makes an object appearing at different places from the actual location; another is called combiner that makes two objects at separated locations looking like only one object.

1. Introduction

It is well known that natural materials with nonuniform refractive index can bend light wave and distort the perception of space, thus creating an erratic phenomenon, such as a mirage in deserts [1]. However, to achieve the desired illusion effects, the material have to be designed in an extraordinary way [2].

Owing to the recent fascinating advances in the research of coordinate transformation method and the development of metamaterials [3,4], it is realized that artificial media with heterogeneous material parameters can manipulate electromagnetic waves in almost any manner. Accordingly, it is competent at making up amazing illusion devices [5–7]. Among various novel wave-manipulation devices, the invisible cloak is the most significant example that can be regarded as creating an illusion of wave propagation in a virtual empty space. Lai et al. [8] further proposed another impressive illusion device theoretically that describes how a particular object could be optically transformed into another, and Li et al. [9] experimentally demonstrate such illusion-optics device.

The work highlighted above lay in the realm of optical transformation, but extensions to acoustic illusion devices were possible. The acoustic interior cloak and exterior cloak, as kinds of illusion devices, have already been proposed [1,10–19]. Acoustic cloaking devices can render objects invisible from the incoming sound, so that they could be used to shield the noise from a building or hide an object from active sound sensing, by changing their scattering characteristics.

In this paper, we demonstrate another new interesting illusion concept, which is different from the acoustic cloaking idea and can change the target's characteristic of radiation so as to make them disguising from the passive sound sensing by

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* Corresponding author. Tel.: +86 10 62796873.
E-mail address: xiangzhihai@tsinghua.edu.cn (Z. Xiang).

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camouflage, i.e., acoustically making an object appearing at different places from the actual location, or making two objects located in different places looking like only one object.

2. Theory of conformal transformation acoustics

As an important branch of "transformation physics", transformation acoustics proposed in [1,11] is based on the invariant of Helmholtz equation under coordinate transformation, which related the virtual space with physical space. This method is noticed to be powerful for designing novel acoustic devices that can manipulate acoustic waves at will. However, exotic material parameters are required in the design from this method, which greatly hampers its practical implementations. In the following we demonstrate that by taking advantage of the conformal mapping, the transformation devices can be realized with an isotropic medium that have very simple material parameters.

The acoustic wave equations in virtual space Ω with coordinate x^i take the form

$$\nabla p = \omega^2 \rho \mathbf{u} \tag{2.1}$$

$$p = -\kappa \nabla \cdot \mathbf{u}, \tag{2.2}$$

where p is the pressure; ω is the angular frequency; $\mathbf{u}$ is the displacement. The fluid medium is isotropic and homogeneous with density ρ and bulk modulus κ.

By eliminating the term $\mathbf{u}$ from (2.1) and (2.2), we have

$$\nabla \cdot \nabla p + \omega^2 \frac{\rho}{\kappa} p = 0. \tag{2.3}$$

According to transformation acoustics, when employing an arbitrary coordinate transformation that maps virtual space Ω into a physical space Ω' with coordinate $x^{i'}$, we have

$$p = p', \quad \mathbf{u} = \mathbf{A}\mathbf{u}', \tag{2.4}$$

where $\mathbf{A}$ is the Jacobian transformation matrix with component

$$A_i^{i'} = \frac{\partial x^{i'}}{\partial x^i}. \tag{2.5}$$

Then the acoustic equations in physical space Ω' can be written as

$$\nabla' p' = \omega^2 \rho \mathbf{u}' \tag{2.6}$$

$$p' = -\kappa \nabla \cdot (\mathbf{A}\mathbf{u}'). \tag{2.7}$$

Moreover, if the transformation $x^{i'}(x^i)$ is restricted to 2D conformal mapping, it is easily seen that

$$\nabla \cdot \mathbf{A} = 0, \tag{2.8}$$

and (2.7) can be expressed as

$$p' = -\kappa \mathbf{A} \nabla \cdot \mathbf{u}' = -\kappa \mathbf{A}\mathbf{A}^{\mathrm{T}} \nabla' \cdot \mathbf{u}'. \tag{2.9}$$

Similarly, by eliminating $\mathbf{u}'$ from (2.6) and (2.9), we have

$$\nabla' \cdot \nabla' p' + \omega^2 \frac{\rho}{\kappa \mathbf{A}\mathbf{A}^{\mathrm{T}}} p' = 0 \tag{2.10}$$

Now, let us denote the conformal mapping as $\zeta(z)$, where

$$\zeta = u + i \cdot v, \quad z = x + i \cdot y \tag{2.11}$$

are complex, it is easy to obtain

$$\mathbf{A}\mathbf{A}^{\mathrm{T}} = |\zeta'(z)|^2. \tag{2.12}$$

Then, comparing (2.3) and (2.10), we can get the material properties of the acoustic device. The density and the bulk modulus in Ω' are written as

$$\rho' = \rho / |\zeta'(z)|, \quad \kappa' = \kappa |\zeta'(z)|. \tag{2.13}$$

Since

$$\sqrt{\rho\kappa} = \sqrt{\rho'\kappa'}, \tag{2.14}$$

the condition of impedance matching between acoustic device and surrounding medium is naturally satisfied.