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Nonexistence of p-Laplace equations with multiple critical Sobolev–Hardy terms

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ABSTRACT

In this paper, we study the nonexistence of solutions for p-Laplace equations with critical Sobolev–Hardy terms and singular terms by using the Pohozaev identity. And the results can be generalized to the case of p-Laplace involving multiple critical Sobolev–Hardy terms and singular terms.

1. Introduction

We consider the following quasilinear elliptic problem:

$$\begin{cases} -\Delta_p u - \mu \dfrac{|u|^{p-2}u}{|x|^p} + \lambda \dfrac{|u|^{p-2}u}{|x-a|^p} = K(x) \dfrac{|u|^{p^*(s)-2}u}{|x|^s} + \gamma |u|^{q-2}u, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases} \tag{1}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$ $(N \geq 3)$, $-\Delta_p u = -\mathrm{div}\left(|\nabla u|^{p-2}\nabla u\right)$, $1 < p < N$, $0 \leq \mu < \overline{\mu} := \left(\frac{N-p}{p}\right)^N$, $a \neq 0$ and 0, $a \in \Omega$, $K(x) \in C^1(\Omega)$. $p^*(s) = \frac{p(N-s)}{N-p}(0 < s < p)$ are critical Sobolev–Hardy exponents. Note that $p^*(0) := p^* = \frac{Np}{N-p}$ is the critical Sobolev exponent. $q \geq p^*$.

In recent years, much attention has been paid to the existence of nontrivial solutions for the quasilinear elliptic problem (1). It is well known that the nontrivial weak solutions for (1) are equivalent to the nonzero

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critical points of the energy functional

$$J(u) =\frac{1}{p}\int_{\Omega}\left(|\nabla u|^p-\mu\frac{|u|^p}{|x|^p}+\lambda\frac{|u|^p}{|x-a|^p}\right)dx-\frac{1}{p^*(s)}\int_{\Omega}K(x)\frac{|u|^{p^*(s)}}{|x|^s}dx-\frac{\gamma}{q}\int_{\Omega}|u|^q$$

defined on $W_0^{1,p}(\mathbb{R}^N)$.

In a pioneering work, Brezis and Nirenberg [1] proved the existence of positive solutions for the nonlinear elliptic problem involving the critical Sobolev exponent. As for the existence of quasilinear elliptic problem with critical Sobolev terms, please see [2]. In [3], Ghoussoub and Yuan considered the existence of positive solutions and sign-changing solutions for elliptic problems with critical Sobolev–Hardy terms, and proved the nonexistence of elliptic problem without Sobolev–Hardy terms by Pohozaev identity.

For elliptic problem with critical terms, Filippucci, Pucci and Robert [4] obtained the existence of positive solutions by Sobolev–Hardy embedding theorem. Others about the research for positive solutions and sign-changing solutions for semilinear elliptic problems, please see [5–8] and references therein.

However, there are few results about the nonexistence of quasilinear elliptic problems with multiple critical Sobolev–Hardy terms and multiple singular points. We investigate the nonexistence of solutions for elliptic problems with multiple critical Sobolev–Hardy terms.

Theorem 1.1. *Assume $\lambda,\ \gamma > 0$ and Ω is a strictly star-sharped domain with respect to the origin in $\mathbb{R}^N$. Moreover*

$$\langle x, \nabla K\rangle \leqq 0, \qquad \langle a, x-a\rangle \leqq 0, \tag{2}$$

for a.e. $x \in \Omega$. Then there is no nontrivial solution for problem (1).

The results in Theorem 1.1 can be easily generalized to the case of p-Laplace equations involving multiple singular points.

Corollary 1.2. *Assume $\lambda_k(1 \le k \le m),\ \gamma > 0$ and Ω is a strictly star-sharped domain with respect to the origin in $\mathbb{R}^N$. Moreover*

$$\langle x, \nabla K\rangle \leqq 0, \qquad \langle b_k, x-b_k\rangle \leqq 0,$$

for a.e. $x \in \Omega$. Then there is no nontrivial solution for problem

$$\begin{cases} -\Delta_p u-\mu\dfrac{|u|^{p-2}u}{|x|^p}+\displaystyle\sum_k\lambda_k\frac{|u|^{p-2}u}{|x-b_k|^p}=K(x)\frac{|u|^{p^*(s)-2}u}{|x|^s}+\gamma|u|^{q-2}u, & x\in\Omega,\\ u=0, & x\in\partial\Omega, \end{cases}$$

where $b_k \in \Omega,\ 1 \leqq k \leqq m$.

The results in Corollary 1.2 can be easily generalized to the case of p-Laplace equations involving multiple critical Sobolev–Hardy terms and multiple singular points.

Corollary 1.3. *Assume $\lambda_k(1 \le k \le m),\ \gamma > 0$ and Ω is a strictly star-sharped domain with respect to the origin in $\mathbb{R}^N$. Moreover*

$$\langle x, \nabla Q_j\rangle \leqq 0, \qquad \langle b_k, x-b_k\rangle \leqq 0, \qquad Q_j(x)\langle c_j, x-c_j\rangle \geqq 0,$$

for a.e. $x \in \Omega$. Then there is no nontrivial solution for problem

$$\begin{cases} -\Delta_p u-\mu\dfrac{|u|^{p-2}u}{|x|^p}+\displaystyle\sum_k\lambda_k\frac{|u|^{p-2}u}{|x-b_k|^p}=\sum_j Q_j(x)\frac{|u|^{p_j-2}u}{|x-c_j|^{t_j}}+\gamma|u|^{q-2}u, & x\in\Omega,\\ u=0, & x\in\partial\Omega, \end{cases}$$

where $0 < t_j < p,\ p_j = p^*(t_j) := \frac{p(N-t_j)}{N-p},\ b_k,\ c_j \in \Omega,\ 1 \leqq k \leqq m,\ 1 \leqq j \leqq n$.