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Solitons and Bäcklund transformation for a generalized (3 + 1)-dimensional variable-coefficient B-type Kadomtsev–Petviashvili equation in fluid dynamics

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ABSTRACT

Under investigation in this paper is a generalized (3 + 1)-dimensional variable-coefficient B-type Kadomtsev–Petviashvili equation, which describes the propagation of nonlinear waves in fluid dynamics. Bilinear form and Bäcklund transformation are derived by virtue of the Bell polynomials. Besides, the one- and two-soliton solutions are constructed via the Hirota method.

1. Introduction

Nonlinear evolution equations (NLEEs) have been regarded as the models to describe some nonlinear phenomena in fluids, plasmas, solid-state materials, etc. [1–4]. Methods have been proposed to address the NLEEs, such as the inverse scattering transformation [5,6], prolongation structure [7], Hirota method [8–10], Darboux transformation [11], Bäcklund transformation [12] and Bell polynomials [13,14].

One of the NLEEs, the Kadomtsev–Petviashvili (KP) equation [15],

$$(\eta_\iota + 6\eta\eta_\varsigma + \eta_{\varsigma\varsigma\varsigma})_\varsigma + \sigma\eta_{\kappa\kappa} = 0, \tag{1}$$

has been seen for the long wave with small amplitude and slow dependence on the transverse coordinate in the single-layer shallow fluid, and surface wave and internal wave in the strait or channel of varying depth

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and width, where $\sigma = \pm 1$, η is the wave-amplitude function of the scaled space coordinates ς, κ and time coordinate ι, while the subscripts denote the partial derivatives.

Extended from Eq. (1), a $(3+1)$-dimensional constant-coefficient B-type KP (BKP) equation has been presented in fluid mechanics [16–18], i.e.,

$$h_{\varrho\varrho\varrho\sigma} + \zeta(h_\varrho h_\sigma)_\varrho + (h_\varrho + h_\sigma + h_\tau)_\varpi - (h_{\varrho\varrho} + h_{\tau\tau}) = 0, \tag{2}$$

where h is a real function of scaled spatial coordinates ϱ, σ, τ and temporal coordinate ϖ, and ζ is a nonzero parameter. For Eq. (2), the one-, two- and multi-soliton solutions [16], conservation laws and some exact solutions [17], Bilinear forms, Bäcklund transformations and some soliton solutions [18] have been discussed respectively.

In the physical situations, on the other hand, when the inhomogeneities of media and nonuniformities of boundaries are taken into account, the variable-coefficient NLEEs can provide some more realistic models than their constant-coefficient counterparts [19–25]. By this token, we will investigate a generalized $(3+1)$-dimensional variable-coefficient BKP equation [25],

$$a(t)u_{xxxy} + \rho a(t)(u_x u_y)_x + (u_x + u_y + u_z)_t + b(t)(u_{xx} + u_{zz}) = 0, \tag{3}$$

which has been proposed to describe the propagation of nonlinear waves in fluid mechanics, where the subscripts mean the corresponding derivatives with respect to the scaled spatial coordinates x, y, z and temporal coordinate t and u is the amplitude or elevation of the relevant wave, while $a(t)$ and $b(t)$ are all the real functions of t and ρ is a real non-zero constant. For Eq. (3), the auto-Bäcklund transformation and shock-wave-type solutions have been obtained [25].

However, to our knowledge, neither the bilinear form and soliton solutions nor the Bäcklund transformation with the Bell polynomials and Hirota method for Eq. (3) have been discussed. In Section 2, concepts and formulas about the Bell polynomials will be introduced. In Section 3, bilinear form, one- and two-soliton solutions for Eq. (3) will be obtained. In Section 4, Bäcklund transformation for Eq. (3) will be presented. Conclusions will be given in Section 5.

2. Bell polynomials

The two-dimensional Bell polynomials are defined as [13,14]

$$Y_{mx,nt}(\phi) \equiv Y_{m,n}(\phi_{i,j}) = e^{-\phi}\partial_x^m \partial_t^n e^{\phi}, \quad (i = 1, \ldots, m, j = 1, \ldots, n), \tag{4}$$

where ϕ is a C^∞ function of x and t, $\phi_{i,j} = \partial_x^i \partial_t^j \phi$, m and n are both the nonnegative integers, while the subscripts in the notation $Y_{mx,nt}(\phi)$ denote the highest-order derivatives of ϕ with respect to x and t. For example,

$$Y_{x,t} = \phi_{x,t} + \phi_x \phi_t, \qquad Y_{2x,t} = \phi_{2x,t} + \phi_{2x}\phi_t + 2\phi_{x,t}\phi_x + \phi_x^2 \phi_t, \ldots. \tag{5}$$

Based on the expressions above, the binary Bell polynomials are derived as [13,14]

$$\mathscr{Y}_{mx,nt}(v,w) \equiv Y_{mx,nt}(\phi) = (\phi_{1,1}, \ldots, \phi_{1,n}, \ldots, \phi_{m,1}, \ldots, \phi_{m,n})\Big|_{\phi_{i,j} = \begin{cases} v_{i,j}, & \text{if } i+j \text{ is odd,} \\ w_{i,j}, & \text{if } i+j \text{ is even,} \end{cases}} \tag{6}$$

where v and w are both the C^∞ functions of x and t, $v_{i,j} = \partial_x^i \partial_t^j v$ and $w_{i,j} = \partial_x^i \partial_t^j w$. For example,

$$\mathscr{Y}_x(v,w) = v_x, \qquad \mathscr{Y}_{2x}(v,w) = w_{2x} + v_x^2, \tag{7a}$$

$$\mathscr{Y}_{x,t}(v,w) = w_{x,t} + v_x v_t, \qquad \mathscr{Y}_{3x}(v,w) = v_{3x} + 3v_x w_{2x} + v_x^3, \ldots. \tag{7b}$$

The link between the $\mathscr{Y}$-polynomials and Hirota D-operators can be defined as [8–10]

$$\mathscr{Y}_{mx,nt}\big[v = \ln(G/F),\, w = \ln(GF)\big] = (GF)^{-1} D_x^m D_t^n G \cdot F, \tag{8}$$