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A variational property of critical speed to travelling waves in the presence of nonlinear diffusion

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ABSTRACT

Let f be a continuous function in $[0,1]$ with $f(0) = 0 = f(1)$ and $f > 0$ on $]0,1[$. We show that, under additional mild conditions on f, the minimal speed for travelling waves of

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[\left|\frac{\partial u}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right] + f(u), \tag{0.1}$$

may be computed via a constrained minimum problem which in turn is related to the solution of a singular boundary value problem in the half line.

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1. Introduction

Throughout this note, let $f : [0,1] \to \mathbb{R}$ be a continuous function such that $f(0) = f(1) = 0$ and $f(u) > 0$ if $u \in (0,1)$. In the theory of Fisher–Kolmogorov–Petrovskii–Piskunov (FKPP) equations, such a function is sometimes referred to as a function of *type A* (see e.g. [1]).

Also, let $p > 1$.

In [2] the notions of admissible speed and critical (i.e. minimal) speed have been introduced for travelling waves to reaction–diffusion equations driven by the one-dimensional p-Laplacian operator, namely

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[\left|\frac{\partial u}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right] + f(u). \tag{1.2}$$

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The relevant front wave profiles $u(x+ct)$ with speed c are given by the (monotone) solutions of the second order problem

$$(|u'|^{p-2}u')' - cu' + f(u) = 0, \quad u(-\infty) = 0,\ u(+\infty) = 1. \tag{1.3}$$

Let q be the conjugate of p, that is $\frac{1}{p} + \frac{1}{q} = 1$. The solutions of the parametric first order boundary value problem (where we write $y_+ = \max(y, 0)$)

$$y' = q(c\,{y_+}^{\frac{1}{p}} - f(u)), \quad 0 \le u \le 1, \qquad y(0) = 0 = y(1), \quad y > 0 \text{ in }]0.1[\tag{1.4}$$

yield the trajectories of solutions of (1.3) via the relationship

$$u' = y(u(t))^{1/p}.$$

We recall the following assumptions, used in [2].

$$M = M_p := \sup_{0<u<1} \frac{f(u)}{u^{q-1}} < +\infty; \tag{H_p}$$

$$\mu := \lim_{u\to 0^+} \frac{f(u)}{u^{q-1}} \text{ exists}, \quad 0 \le \mu < +\infty. \tag{H_p'}$$

It follows from results in [2] that there is a 1–1 correspondence between solutions of (1.3) (up to translation) taking values in $]0,1]$ and solutions of (1.4) that are strictly positive in $]0,1[$. These sets of solutions are nonempty provided (H_p) holds. Also, basic properties of the profiles and their speeds, now classical in the FKPP theory ($p = 2$), were extended in [2] to the p-Laplacian model. In particular, if (H_p) holds, the set of admissible speeds – that is, values of the parameter c such that (1.4) has a solution – is an interval $[c^*, +\infty[$ where

$$\mu^{\frac{1}{q}} p^{\frac{1}{p}} q^{\frac{1}{q}} \le c^* \le M^{\frac{1}{q}} p^{\frac{1}{p}} q^{\frac{1}{q}} \tag{1.5}$$

(the first inequality being valid if the stronger (H_p') holds). The minimum admissible value c^* of the parameter c is called *critical speed*.

Remark 1.1. An elementary calculation on the basis of (1.4) shows that, given a number $a > 0$, c is an admissible speed with respect to f if and only if $ca^{\frac{1}{p}}$ is admissible with respect to af.

For the case of linear diffusion ($p = 2$), variational characterizations of the critical speed c^* are known: in [3] a variational formulation is presented, based on the second order ordinary differential equation satisfied by the wave profiles; in [4] the authors use the first order model that represents the wave trajectories in a phase plane to establish another defining property of variational type for c^*.

The purpose of this note is to obtain a variational property of c^* in the framework of (1.3). We shall use some ideas from [3].

Remark 1.2. It will be useful for our purpose to recall the role played by functions of type B. A function $f : [0,1] \to \mathbb{R}$ is said to be of type B if it is continuous and there exists $\delta \in]0,1[$ such that $f(s) = 0$ if $0 \le s \le \delta$ or $s = 1$, and $f(s) > 0$ if $\delta < s < 1$.

It is known that if f is of type B there exists exactly one admissible speed c^* of (1.3), that is, (1.4) has a positive solution for exactly this value of the parameter c. Moreover, if f_n is a nondecreasing sequence of functions of type B and $\lim_{n\to\infty} f_n(x) = f(x)$, then with obvious notation $\lim_{n\to\infty} c^*(f_n) = c^*(f)$. See [2], section 4.

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